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TECHNICAL PAPERS

DISCUSSIONS

APPLICATIONS FOR ADMISSION AND TRANSFER

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MEMBERSHIP

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

TESTS FOR HYDRAULIC-FILL DAMS[1]

By Harry H. Hatch,[2] M. Am. Soc. C. E.

Synopsis

This paper describes some of the tests made for the City of Springfield, Mass., on the Cobble Mountain Hydraulic-Fill Dam. The derivation is given of almost all the formulas used in the paper. In addition to new formulas and data pertaining to tests, the paper contains a consolidation study of core materials and a seepage formula for hydraulic-fill dams. It also gives the results of the gradation tests made at Cobble Mountain Laboratory of the core materials of several other hydraulic-fill dams.

Its main object, however, is to arouse interest among engineers connected with hydraulic-fill construction, with a view of standardizing methods of testing materials in hydraulic-fill dams. Opinions differ; engineers on each job devise a different method of testing; and it is time that a definite step is taken and a standard method developed in making tests for this particular kind of construction.

The original manuscript, which is several times the length of this paper, is on file for reference in Engineering Societies Library, 33 West 39th Street, New York, N. Y. It contains forty-one illustrations, including curves of many of the equations.

Purpose of the Laboratory

Hydraulic-fill dams are almost new compared with other types, and they are becoming more popular because of their many advantages. Unfortunately, methods of testing materials for such dams have not been standardized. No doubt the materials for different dams at different locations are not the same, and many dams have local problems; but in the main a standard method could be devised, such as is being used for cement and other materials of construction, and, whenever necessary, this method could be modified for

Note.—Discussion on this paper will be closed in January, 1933, *Proceedings*.

[1] Presented at the Joint Meeting of the Irrigation and Power Divisions, Yellowstone National Park, Wyoming, July 7, 1932.

[2] Engr. in Chg., Cobble Mountain Reservoir, Springfield Water-Works, Westfield, Mass.

local conditions. After all, the requirements of the core and beaches of a hydraulic-fill dam are the same for any dam and at almost any location.

Heretofore, the methods of testing the borrow and core materials of hydraulic-fill dams have not been along the same lines. As a rule, the ordinary sieve analyses have been made with the statement that a certain percentage of the material is finer than a 100-mesh or 200-mesh sieve. Often the sieves are not even rated; for sieves finer than the 200 mesh the methods differ greatly, and as could be expected, the results are not always entirely reliable. The purpose of Cobble Mountain Laboratory was to make all the tests necessary for this kind of construction and also to establish a satisfactory, and yet practical, method of grading the material finer than the 200-mesh sieve to at least 0.01 mm.

The fact that the laboratory was established on the job proved of great advantage in keeping a close control of the borrow and sluiced materials of the dam.

Sampling

In making the preliminary studies on borrow-pits for the determination of the percentage of a critical size and also for making the gradation curve of the borrow material, one must have a representative sample of the pit. Since the gradation curve must show the material that will go over to the dam, it should not include the large boulders in the pit that will be left behind.

Particular care and special equipment are required to secure representative samples from the core of a dam during its construction. At Cobble Mountain the so-called projectile sampler, designed by the Staff of Murray and

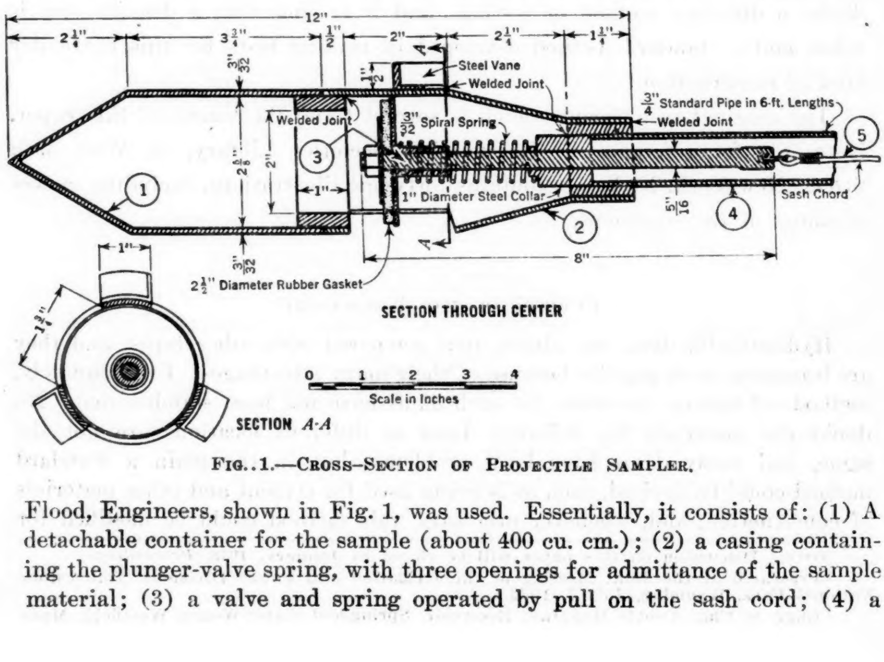

FIG. 1.—CROSS-SECTION OF PROJECTILE SAMPLER.

Flood, Engineers, shown in Fig. 1, was used. Essentially, it consists of: (1) A detachable container for the sample (about 400 cu. cm.); (2) a casing containing the plunger-valve spring, with three openings for admittance of the sample material; (3) a valve and spring operated by pull on the sash cord; (4) a

standard, $\frac{3}{4}$-in. pipe; and (5), the sash cord extending to the surface of the core pool. The sample, taken out with the projectile sampler from a more or less consolidated core, will be of no value for the determination of moisture content or percentage of voids. It is almost impossible to bring out the sample in its original condition with this device.

The following method was found to be satisfactory for taking out samples in their original condition from a consolidated core. One 2-in., or one $1\frac{1}{2}$-in. pipe (or a series of them coupled to any length), having the bottom covered with a canvas, was lowered to a desired depth by means of additional weights. After making sure that there was no water at the bottom of the pipe, a $\frac{3}{4}$-in. pipe was lowered inside the larger one. The smaller pipe had a piece about 1 ft. long at the bottom, which was beveled to a cutting-edge. The smaller pipe was forced down (the cutting-edge piercing the canvas) into the core about 1 ft., or more. Then it was raised, and its lower end section was uncoupled. The sample was forced out by a wooden piston, and was immediately brought to the laboratory where the moisture content and percentage of voids were determined as soon as possible.

Specific Gravity

The specific gravity of the materials is of great importance. After the value of the specific gravity is determined definitely, it should be checked occasionally to see whether there is any variation. The tests at Cobble Mountain Dam were made essentially as described[3] in the method for testing cement of the American Society for Testing Materials.

The Percentage of Voids

The percentage of voids and moisture content are related, and they give a good idea of the consolidation of the core. The late Allen Hazen, M. Am. Soc. C. E., stated[4] that,

> "The percentage of voids furnishes, on the whole, the best index of consolidation and stability. In itself, it is not an adequate basis of comparison, because different kinds of core material may have different degrees of stiffness with the same percentage of voids; but, notwithstanding this difference, the percentage of voids is the best index of consolidation so far available."

The determination of the percentage of voids, if it is done accurately, is essential. It is readily seen that the unit weight (and, therefore, the acting and resisting forces) and the coefficient of permeability of the different parts of a dam are directly dependent on, and the coefficient of friction is affected by, the percentage of voids of its material.

If at any depth and location in the core a sample in its original position with its original moisture content can be brought to the surface, the moisture content of the core in the dam at that particular elevation can easily be computed. In saturated core material all voids are filled with water and,

[3] Specifications C 9–26, Am. Soc. for Testing Materials.

[4] "Hydraulic-Fill Dams", by Allen Hazen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1718.

when the specific gravity of the material is known, the percentage of voids can be computed by the formula,

$$V = \frac{w}{w + \frac{(100 - w)}{s}} = \frac{ws}{ws + (100 - w)} \qquad (1)$$

in which, V equals percentage of voids; w, percentage of water by weight; and, s, specific gravity of core material. The computation is simple and can be made readily, or the equation can be interpreted into a set of curves from which values can be picked.

Two methods were used in the determination of the percentage of voids in the beach materials—the one for the beach material near the core limits, where the particle sizes are small, and the other for the beach material near the toes, where the particle sizes are large.

For beach materials with particle diameters up to about $\frac{1}{4}$-in., brass cylinders, 4 in. in diameter and 6 in. long, were used. One end of the cylinder was beveled to a cutting-edge to facilitate forcing it into the beach. Each cylinder was pressed into the beach and then removed with a shovel having considerable material above and below it. The top and bottom of the sample were neatly cut with wire, giving a sample equal in volume to that of the cylinder. The material was then dried and weighed, and the percentage of voids computed.

For larger particles near the toes of the beach, a steel pipe, 20 in. in diameter and about 3 ft. long, open at both ends, was used. At places where the samples were taken the surface material was scraped down to a level and undisturbed section in the beach. The pipe was forced down by sledge hammer only 4 or 5 in. at a time; then the inside material was cleaned out and set apart. Later, it was forced down a similar distance, and the operations continued until about 4 cu. ft. of sample material was obtained. After it came out of the pipe, the test material was weighed in the field and a sample was brought to the laboratory for moisture-content determination. With this information the dry weight of the sample in the field was computed and the percentage of voids determined.

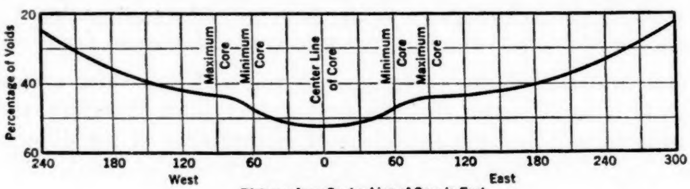

FIG. 2.—PERCENTAGE OF VOIDS ON A SECTION OF COBBLE MOUNTAIN DAM AT DIFFERENT DISTANCES FROM THE CENTER LINE OF THE CORE.

It was noted that the percentage of voids was at its maximum in the core pool, decreasing toward the toes (Fig. 2). The reason for this may be due to the fact that the beach material is deposited by running water and that the core material held in suspension is deposited by settling in comparatively still water. Furthermore, the material close to the core limits is more or

less uniform in size, whereas farther away from the core section the beach material becomes larger, has a better gradation, and, therefore, a smaller percentage of voids.

The percentages of voids shown on Fig. 2 are not final and are good only as a comparison in showing the variation in consolidation of the various points in the dam at or about the same elevation. Undoubtedly, due to increased weight above, the consolidation shown in Fig. 2 will be somewhat increased as the dam is completed and as the final settlement takes place.

Penetration Tests

A method previously used has been to make the penetration tests in the field by two strong men forcing down one $1\frac{1}{2}$-in. pipe into the core as far as it will go. It has been stated that when the pipe is forced down, the core material at the elevation of the bottom of the pipe has 50% voids. The definition of "two strong men" is rather uncertain, and it is doubtful whether 50% voids could be obtained for any two materials at the elevation of the bottom of the pipe, even if the force exerted by "two strong men" was constant and known. Experience shows that the penetration of the pipe is affected not only by the percentage of the voids in the core, but also by the grain sizes and other physical characteristics of the core material.

To obviate any uncertainty in regard to the force necessary to reach the penetration limit on the $1\frac{1}{2}$-in. pipe, definite loads up to 600 lb. were applied to sink it. A complete operation was conducted for each loading. In some cases, the bottom of the pipe was tied with greased canvas to prevent mud and water from getting into it. However, it was found that the depth of penetration was not materially affected by the canvas at the bottom of the pipe. When the penetration limit was reached for any loading, the weights were removed, and samples were taken from the core by the $\frac{3}{4}$-in. pipe, as described under "Sampling," at the elevation of the bottom of the $1\frac{1}{2}$-in. pipe, and the percentage of voids of the material was computed.

It was discovered that by applying the total load at the beginning of the test, the pipe would penetrate farther into the core than by applying the same total load in fractions at different intervals during the process of penetration, when it took time to make the necessary additional connections and to increase the loading. In every instance, the latter case gave about 2 or 3 ft. less penetration.

It is evident that there is a distinct relation for the same material between the total load applied to the $1\frac{1}{2}$-in. pipe and the percentage of voids of the core material at the elevation of the bottom of the pipe. Once this relation is established for any one material, it will not always be necessary to determine the percentage of voids for every test.

Notwithstanding the personal equation both in the field and in the laboratory, the difference in physical characteristics, and the difference in the grain sizes of the core material, the numerous tests gave the following approximate relation between the percentage of voids of the material and the total load causing the penetration:

$$100 - V = 33\, L^{0.067} \qquad (2)$$

in which, V is the percentage of voids, and L, the load, in pounds, including the weight of the $1\frac{1}{2}$-in. pipe. As in the case of Equation (1) this formula can easily be expressed in curve form for easy reference. In winter, it was more convenient to carry on this investigation by building a timber tower over the ice on the core pool, in order to facilitate the operations necessary for conducting the test.

Rating Sieves

The mesh openings of a sieve given by the manufacturer, or their exact measurement, will not give the actual separation sizes of the material. Certain rules have been established to apply to the nominal mesh openings of a sieve in order to obtain its actual separation; but it is desirable to determine the actual separation of the sieves used on the job for the particular material on hand. The shape and other characteristics of a material will affect the value of the separation. A sieve will not necessarily have the same separation size for different materials. In fact, rating of the same set of sieves should be repeated from time to time as the meshes will wear and some of the finer meshes will get clogged.

The 200-mesh sieve is the most unreliable in this respect; it requires constant watching. The rating of sieves on the Cobble Mountain Dam was

TABLE 1.—The Variation in Separation Sizes of the Tyler Sieves Used at Cobble Mountain

Mesh No.	Mesh opening, in millimeters (Tyler values)	Separation, in Millimeters, by Ratings					Separation values used
		No. 1	No. 2	No. 3	No. 4	No. 5	
4	4.699	5.02	5.03	....	....	....	5.02
8	2.362	2.60	2.60	....	....	....	2.60
14	1.168	1.29	1.29	....	1.27	....	1.27
28	0.589	0.65	0.63	0.647	0.62	....	0.62
48	0.295	0.33	0.32	0.372	0.31	....	0.31
100	0.147	0.16	0.16	0.166	0.15	....	0.15
200	0.074	0.086	0.082	0.086	0.095	0.09	0.09

conducted along the principles outlined[5] by the American Water Works Association. The variation in separation sizes is shown in Table 1. The No. 3 rating was checked by microscopic examination, and its variation was found to be due to differences in the shapes of particles.

Elutriation

A sieve analysis with a minimum mesh of 200 is limited to about 0.09 mm. in diameter, whereas it was necessary to carry an analysis to at least 0.01 mm., or about one-tenth the particle sizes of the 200-mesh limit.

Elutriation by the decantation method, as described[6] by F. E. Turneaure and H. L. Russell, Members, Am. Soc. C. E., has been used to analyze sand that passes a 200-mesh sieve. The tests at Cobble Mountain Dam were made to control the sizes of the core material during construction. The method cannot be practical because of the lack of automatic control in operations

[5] Manual of Water Works Practice, 1925, p. 641.

[6] "Public Water Supplies", by F. E. Turneaure and H. L. Russell, 1910, p. 472.

and because of the many personal equations entering into the test, not to mention the slow process. Hence, it became necessary to develop a practical method that would take a reasonably short time to make tests of a few samples a day with reliable results, and, therefore, a practical apparatus for elutriation was designed, and grading by elutriation was adopted.

Various types of elutriators[7] have been used in laboratories for some time, some of them dating back to about 1900. For the first time, at Cobble Moun-

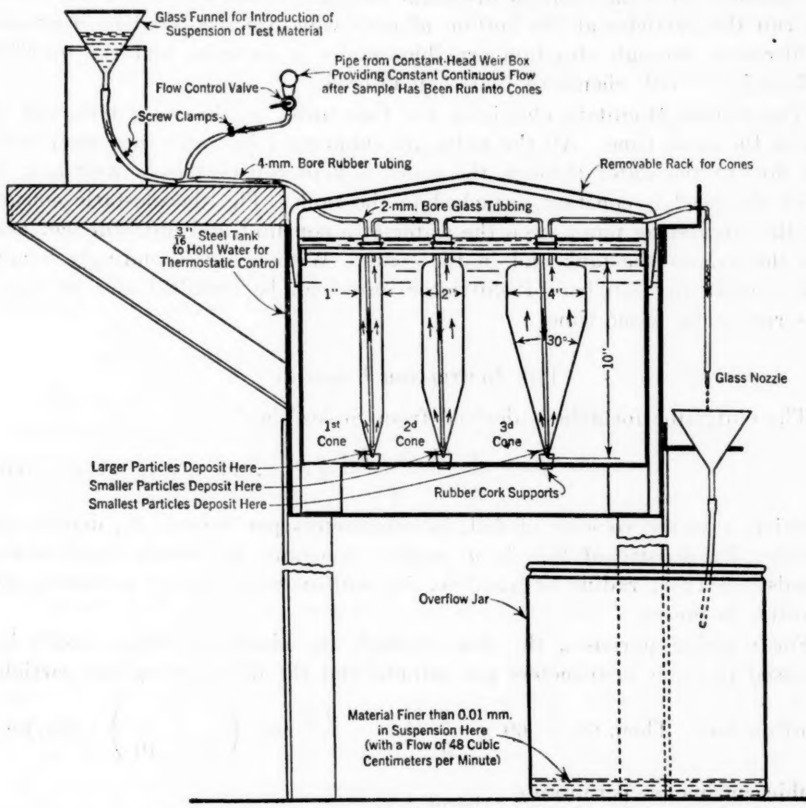

FIG. 3.—ONE UNIT OF ELUTRIATOR TO SHOW OPERATION.

tain Dam, an elutriation method was used in a practical way for the control of the core material of a dam under construction.

The apparatus (see Fig. 3) consists of three glass cones of different sizes, each provided with an inlet tube running to the bottom and an outlet tube running to the next cone. The first cone inlet tube has a connection to a glass funnel into which the test material is first placed. From this funnel, the material is carried to the first, then the second, and then the third, cone which has a connection to a glass nozzle that discharges the overflow into a large glass jar.

[7] "Clays", by Heinrich Ries, 1927, pp. 198–202.

With the arrangement of the inlet tubes, a rising column of water is created in every cone, each with a different velocity, due to the different cross-sections of the cones. When the sample is washed through the glass funnel into the first cone only the larger particles settle because at this point the water has its greatest velocity. The smaller particles are carried over intc the next cone. Similarly, the same action takes place in the other cones, the particles settled in them having a definite relation to their areas. The test is continued until the overflow discharge becomes almost clear. At the end of the run the particles at the bottom of each cone are recovered by a process of filtration through alundum crucibles under a vacuum, and the overflow is flocculated with chemicals and then filtered.

The Cobble Mountain elutriator has four units, so that four tests can be run at the same time. All the units are submerged in water in a steel tank. The flow of the water through the cones is kept constant by a weir box, in which the head is constant, attached to the tank. The thermostatic control and the circulating pump keep the water at a constant and uniform temperature throughout the tank and in the cones. With a correspondingly longer tank, almost any number of elutriator units can be installed and as many tests run at the same time.

The Elutriator Formula

The elutriator formula is derived from Stokes' law[8]:

$$v = \frac{2}{9} \frac{(P_p - P_l)}{n} g\, r^2 \qquad (3)$$

in which, v is the velocity of fall, in centimeters per second, P_p, density of particle; P_l, density of liquid; g, gravity constant, in centimeter-gramme-seconds = 981; r, radius of particle, in centimeters; and n, coefficient of viscosity, in poises.

For working purposes, the flow through the elutriator tubes should be expressed in cubic centimeters per minute and the diameter of the particle in millimeters. Then, $60v = 60 \times \frac{2}{9} \frac{(P_p - P_l)}{n} 981 \left(\frac{d}{2 \times 10}\right)^2$; but, $60v$ $= \frac{\text{cubic centimeters per minute}}{\text{square centimeters}}$; therefore,

$$\frac{\text{cubic centimeters per minute}}{\text{square centimeters}} = \frac{60 \times 2 \times 981}{9 \times 4 \times 100} \left(\frac{P_p - P_l}{n}\right) d^2$$

$$= 32.7 \left(\frac{P_p - P_l}{n}\right) d^2 \qquad (4)$$

Without appreciable error values of specific gravity instead of those of density can be substituted for P_p and P_l. Let $P_p = 2.72$ for Cobble Mountain material, and $P_l = 1.00$. Then, $P_p - P_l = 1.72$. Substituting this value in

[8] Mathematical and Physical Papers, by G. G. Stokes, Vol. III, p. 60, Cambridge Univ. Press, 1905.

Equation (4) and solving for d,

$$d = 0.13334 \sqrt{\frac{\text{cubic centimeters per minute}}{\text{square centimeters}}} \sqrt{n} \quad \ldots\ldots\ldots\ldots (5)$$

which is the elutriator formula. By inserting the actual values of the flow, area of the tube, and the coefficient of viscosity, in Equation (5), the diameter of separation can be computed in millimeters.

Instead of the viscosity coefficient, it is proposed to use the temperature factor, $\frac{60}{t + 10}$, in degrees Fahrenheit. The relation between the coefficient of viscosity and $\frac{60}{t + 10}$ is not strictly constant; the relation factor varies from 0.01274 to 0.01311 for a range of temperature from 40° to 90° Fahr. The average value of 0.01296 was adopted to apply to the range of working temperature for this factor; thus,

$$n = 0.01296 \left(\frac{60}{t + 10}\right) \quad \ldots\ldots\ldots\ldots (6)$$

Substituting for n in Equation (5) the value from Equation (6) and making the necessary reduction,

$$d = 0.0152 \sqrt{\frac{\text{cubic centimeters per minute}}{\text{square centimeters}}} \sqrt{\frac{60}{t + 10}} \quad \ldots\ldots\ldots\ldots (7)$$

Equation (7) is the one used at Cobble Mountain Reservoir. Eleven microscopic measurements were made at the hydraulic-fill laboratory, and their values for the constant, 0.0152, in Equation (7) were: 0.0155, 0.0154, 0.0172, 0.0163, 0.0157, 0.0155, 0.0157, 0.0151, 0.0165, 0.0151, and 0.0161. The average of these eleven values is 0.0158 compared with 0.0152 as computed. The difference, which is probably due to shape of particles, is negligible; and the two results agree satisfactorily.

Stokes' law gives the constant velocity of falling bodies with a certain diameter through a liquid, whereas, in this case, the same law applies to bodies of the same diameter held in suspension by, or resisting the upward flow of, the same liquid, having, of course, the relative values given in Equation (7).

In order to obtain reliable results for a test, it is necessary that the flow (in cubic centimeters per minute), and the temperature (t° Fahr.) factors of Equation (7) should remain constant during the run of the test. The weir box furnishes a constant head, and a desired uniform flow can be had throughout the run from a small quantity to about 80 cu. cm. per min. The heat control can be set to a desired temperature, and it will work automatically to keep the water at that constant temperature. The circulating pump, of course, will insure a uniform temperature for the water throughout.

The net cross-sectional areas of the cones are: $A_1 = 4.65$; $A_2 = 20.81$; and, $A_3 = 85.00$ sq. cm. With a flow of 48 cu. cm. per min. and $t = 77°$ Fahr., Equation (7) will give, to nearest 0.01 mm.:

$d_1 = 0.04$ mm. for first stage, the smallest cone.
$d_2 = 0.02$ mm. for second stage, the middle cone.
$d_3 = 0.01$ mm. for third stage, the largest cone.

It is desirable to keep the separation sizes constant; and in case of any change in the temperature, the flow can be adjusted to offset this temperature effect, thus giving a constant, *d,* in Equation (7). It is seen from the formula that *d* varies as the square root of the flow. Then, by adjusting the flow to desired values, separation can be determined for corresponding smaller sizes.

As a practical procedure, however, this is not advisable. Duration of the test varies inversely as the flow. A flow of 48 cu. cm. per min. will give a lowest separation of 0.01 mm., and the actual run of the test through the elutriator will take nearly four times as long for a separation to a fineness of 0.005 mm. Of course, according to the formula, the law will also hold for higher flows—no doubt within limits—with larger separations and less time.

The results obtained by the elutriator are reliable especially when they are checked now and then by microscopic examination. This method is a great improvement in the accurate and rapid determination of grain sizes finer than the 200-mesh sieve compared with other methods used heretofore.

Microscopic Control

Microscopic measurements may involve assumptions which perhaps are not exact and which may influence the results, and yet it is the most reliable method available for observation and checking purposes. The microscope is very useful also to observe and study the shape and other characteristics of particles.

Periodically, microscopic measurements of the separation of the 200-mesh sieve and also of the three stages of the elutriator were made at Cobble Mountain in accordance with the method outlined[9] by Professor Lincoln T. Work.

The Hydrometer

The most rapid method, with reasonable accuracy, in determining the grain sizes finer than the 200-mesh sieve, known to the writer, is by means of the hydrometer developed by Dr. J. G. Bouyoucos.[10] Within almost an hour, one-third of the minimum time necessary for the elutriator, an analysis can be made of a soil sample by this method. The apparatus required is very simple and its cost is almost negligible compared with that of the elutriator. This method has many advantages, and it is being used in many places as the only means of determining the grain sizes of soil particles.

The hydrometer is so constructed that the graduation on its stem at the water surface will indicate the number of grammes of soil in suspension per liter of water at the center of bouyancy of the hydrometer. As a precaution, however, the readings should be checked with various predetermined quantities of grammes of soil per liter of water, or with solutions of known con-

[9] "Methods of Particle Size Determination", by Lincoln T. Work, *Proceedings*, Am. Soc. for Testing Materials, Vol. 28, Pt. II, 1928, p. 780.

[10] "Directions for Determining the Colloidal Material of Soils by the Hydrometer Method", by J. G. Bouyoucos, *Science*, July 1, 1927, Vol. LXVI, No. 1696, pp. 16–17; and "Making Mechanical Analyses of Soils in Fifteen Minutes", by J. G. Bouyoucos, *Soil Science*, Vol XXV, No. 6, June, 1928, pp. 473–480.

centration. The theory upon which this method is based follows also from Stokes' law as given by Equation (3). Substituting the values of $P_p = 2.72$, $P_l = 1.00$, $g = 981$, and $r = \frac{d}{2}$,

$$d = 0.1033\ (vn)^{\frac{1}{2}} \qquad (8)$$

Equation (8) can also be used for d, in millimeters, with n, in centipoises. The value of n for any temperature can be obtained from tables of coefficients of viscosity; therefore, it only remains to find the value of v for the determination of d; thus,

$$v = \frac{\text{Distance, in centimeters}}{\text{Time, in seconds}} \qquad (9)$$

At different time intervals observations are made of the concentration (in grammes of soil per liter of water) of the suspension at the hydrometer center of buoyancy. From the beginning of the test to the time of any observation, during so many seconds, particles having diameters larger than a certain critical size are traveling by, or particles finer than that critical size are still in suspension above, the center of buoyancy. The traveled distance is

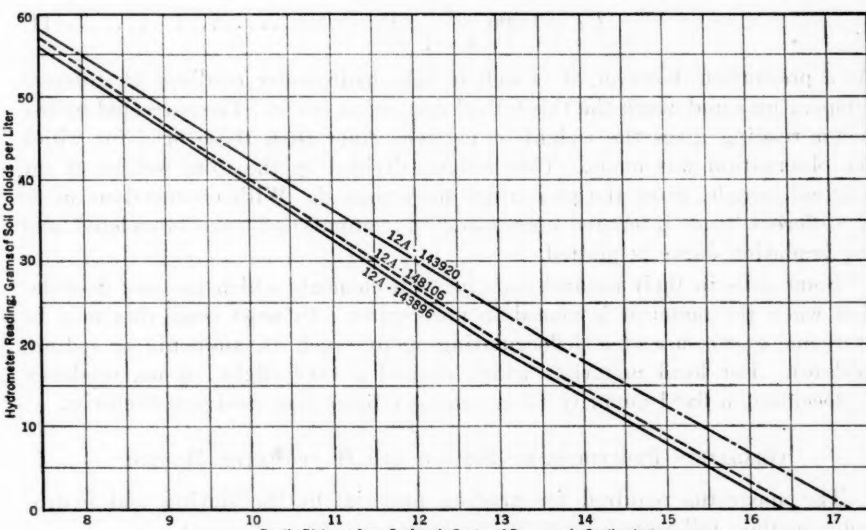

FIG. 4.—CURVES SHOWING DISTANCE FROM CENTER OF BUOYANCY TO WATER SURFACE, PLOTTED AGAINST GRAMMES OF SOIL PER LITER.

taken from the water surface to the buoyancy center, but this is constant for different concentrations of a suspension; neither is it necessarily the same for any two hydrometers. For each hydrometer a curve similar to Fig. 4 should be prepared. With distances obtained from Fig. 4, and the time, in seconds, as observed, the velocity is computed and Equation (8) is solved for d.

The hydrometer has been designed for a water temperature of 67° Fahr. = 19.44° cent. If water is used with another temperature it will be necessary to make a correction due to its change in density according to the formula,

$$R_0 = \frac{s}{s-1}\,1\,000\,(P_0 - P_t) + R_t \quad\text{.....(10)}$$

in which, R_0 is hydrometer reading at 19.44° cent.; s, specific gravity of soil; P_0, density of water for 19.44° cent.; P_t, density of water at $t°$ cent.; and R_t, hydrometer reading at $t°$ cent.

The term, $\frac{s}{s-1}\,1\,000\,(P_0 - P_t)$, is the correction to be applied to the reading of the hydrometer at any temperature. The water has a maximum density at about 4° cent., and it decreases gradually as the temperature rises. The density of water below 4° cent.—in fact, below that of the working temperature—need not be considered. Whenever the temperature is below 19.44° cent., the correction will be negative, or with a minus sign, and *vice versa.* The correction term can be plotted for any value of specific gravity, from the relation:

$$C_t = 1\,000\,\frac{s}{s-1}\,(P_0 - P_t) \quad\text{.....(11)}$$

As a precaution, however, it is well to take hydrometer readings at different temperatures and check the theoretical correction curve. The corrected hydrometer reading gives the weight of particles finer than those of d for which the observation was made. This weight, divided by the total weight of the original sample, gives the percentage finer than d. With observations made at different time intervals, corresponding computations are completed and the gradation curve is plotted.

Some soils in their natural state contain elements which produce flocculation when the material is placed in pure water. In most cases this may be overcome by the use of a deflocculating agent (such as, ammonia or sodium oxalate). For local materials which showed a very slight, or no, tendency to flocculate, a fixed quantity (5 cu. cm. per liter) was used satisfactorily.

Gradation Procedure by Sieving and Hydrometer Method

The apparatus required for grading material by the sieving and hydrometer method follows:

- One gyratory riddle with nest of Tyler sieves (Mesh Nos. 4, 8, 14, 28, 48, 100, and 200);
- One 250-mesh sieve;
- One motor, cup, and stirring device;
- One glass cylinder graduated to about 1 100 cu. cm.;
- One hydrometer graduated in grammes of soil colloids per liter of water at 67° Fahr.;
- One centigrade thermometer (—10° to 110° cent.);
- One stop-watch; and miscellaneous laboratory equipment, including wash bottle, balance, drying oven, etc.

The preliminary treatment entails five steps, as follows:

1.— Determine the moisture content of the moist soil sample to be tested, and weigh out the amount necessary to give about 35 grammes of —200-mesh material.
2.— Place the sample in the stirring cup, washing out all particles from the weighing container with water; and fill the cup about three-fourths full with water and add 5 cu. cm. of ammonia or sodium oxalate ($Na_2C_2O_4$). Experiments with local materials showed that the 5 cu. cm. of dispersing agent gave satisfactory results in all cases.
3.— Place the cup in the stirrer, and stir for about 10 min.
4.— Wash the material through a 250-mesh sieve, drying the coarse fraction at 105° cent.
5.— Make a sieve analysis of the coarse portion (Sieve Nos. 4, 8, 14, 28, 100, and 200).

The sample is prepared for the hydrometer run in two steps, namely,

6.— Add the —200-mesh material from the dry sieving to the —250-mesh material in suspension from the preliminary washing; and
7.— Transfer the suspension to a liter graduate and fill with water to a volume equivalent to 1 000 cu. cm. plus the volume of soil used.

Then, the hydrometer run involves four final steps:

8.— Agitate the suspension by reversing the cylinder about ten times, covering the opening with the palm of the hand.
9.— Set the cylinder on a table and start the stop-watch immediately.
10.—Place the hydrometer in the cylinder.
11.— Read the hydrometer at 1, 2, 5, 10, 30, and 60 min., taking care to keep the hydrometer stem wet so that observations are taken on similar menisci.

Computation.—A computation based on an analysis of Sample *P*-11, Test *H*416, will be used to illustrate this procedure:

Moisture Content, by Step 1:

		Grammes.
Wet material + crucible	=	23.790
Crucible empty	=	12.874
Wet material	=	10.916
Dry material + crucible	=	20.954
Crucible empty	=	12.874
Dry material	=	8.080

$$\text{Percentage of dry material} = \frac{8.080}{10.916} \times 100 = 73.9$$

A total of 80 grammes of wet material was used; therefore, its equivalent dry weight was 80 × 0.739 = 59.12 grammes.

Sieving, by Step 5:

The weight of the + 250-mesh material to be sieved is 40.63 grammes. The analysis of the sample given in Table 2, shows that 40.57 grammes were recovered in the sieving. In other words, 40.63 − 40.57 = 0.06 grammes, lost in sieving.

TABLE 2.—ANALYSIS OF MATERIAL RETAINED ON A 250-MESH SIEVE

Mesh No.	Separation, in millimeters	Grammes retained	Cumulative grammes finer	Cumulative percentage finer
4..............	5.02	0	.....	100.0
8..............	2.60	0	.....	100.0
14..............	1.27	0	59.06	100.0
28..............	0.62	0.05	59.01	99.9
48..............	0.31	0.20	58.81	99.6
100..............	0.15	3.20	55.61	94.1
200..............	0.09	10.03	45.58	77.1
200 (dry)...........	....	27.09	.....	.....
Total sample recovered from sieving........	....	40.57	.....	.....

Hydrometer Run, Steps 8 to 11, Inclusive:

The arrangement of observations and computations for Steps 8 to 11, inclusive, is shown in Table 3.

TABLE 3.—OBSERVATIONS AND COMPUTATIONS

Elapsed time, in minutes	Temperature, in degrees centigrade	HYDROMETER READING		Distance, in centimeters	Velocity, in centimeters / seconds	Viscosity, n	Diameter, in millimeters	Percentage finer
		Observed	Corrected					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	19.2	27.3	27.2	11.95	0.199	1.025	0.0467	46.0
2	19.2	19.8	19.7	13.10	0.109	1.025	0.0345	33.3
5	19.2	12.0	11.9	14.45	0.0482	1.025	0.0230	20.2
10	19.2	9.0	8.9	14.95	0.0249	1.025	0.0165	15.1
30	19.0	6.0	5.9	15.50	0.0089	1.030	0.0097	10.0
60	19.0	4.3	4.2	15.80	0.0044	1.030	0.0069	7.1

These values (Table 3) are computed, as follows:

Column (1)..........	Observed
Column (2)..........	Observed.
Column (3)..........	Observed.
Column (4)..........	From Equation (10).
Column (5)..........	From Column (3), Table 3, and Fig. 4.
Column (6)..........	From Columns (1) and (5), Table 3; and Equation (9).
Column (7)..........	From Fig. 5.
Column (8)..........	From Equation (8).
Column (9)..........	Column (4) Table 3, divided by total sample (59.06 grammes).

Plotting.—Plot the hydrometer grading, together with the sieve analysis showing the particle size, in millimeters, against the cumulative percentage passing a given size of sieve. (See Fig. 6.) From the grading curve pick off the 10% (effective) size, and the 60% size and compute the uniformity coefficient, by dividing the latter by the former.

COMPARISON OF ELUTRIATOR AND HYDROMETER METHOD RESULTS

Many comparisons have been made of the results obtained by the elutriator and the hydrometer methods at Cobble Mountain. The hydrometer gradation curves, in the lower limits, are invariably above those of the elutriator, thus showing a higher percentage of finer particles. (See Fig. 6.) Logically, it

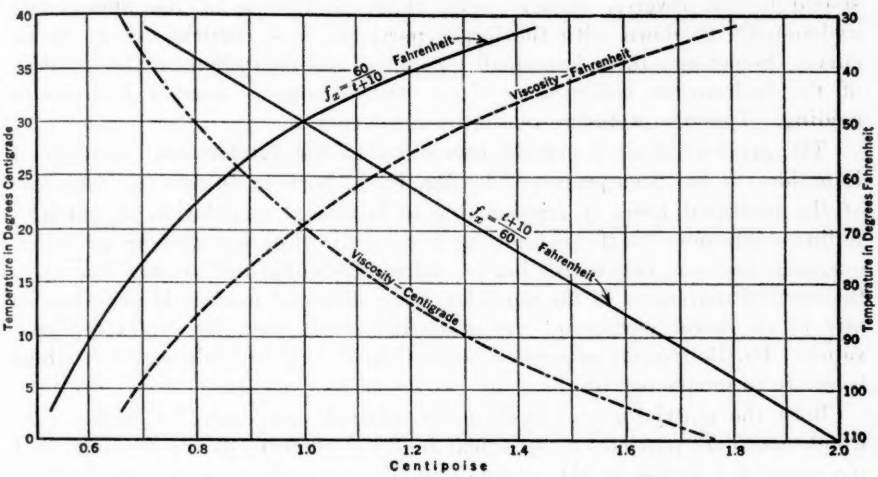

FIG. 5.—RELATION BETWEEN VISCOSITY AND TEMPERATURE FACTOR.

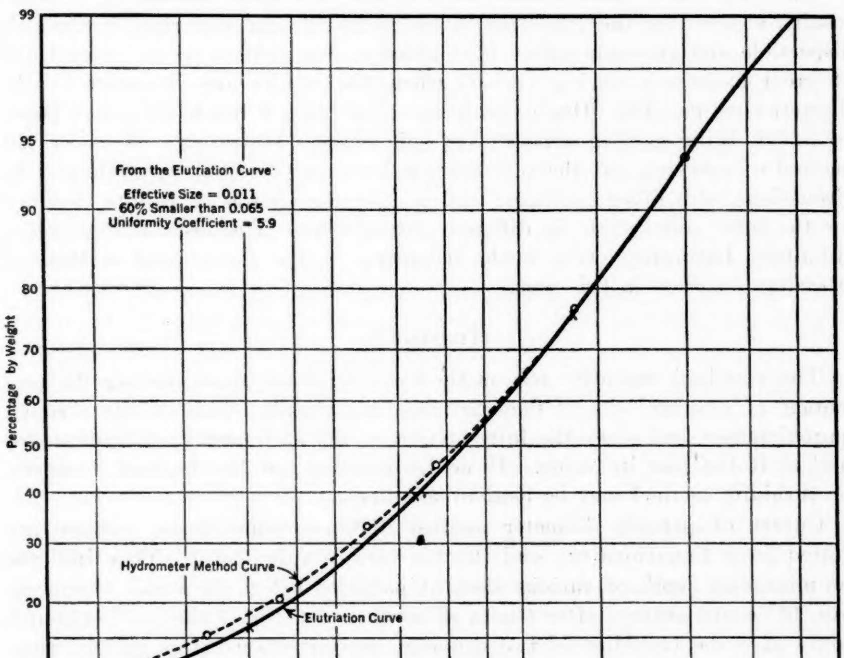

FIG. 6.—COMPARISON OF GRADATION BY THE ELUTRIATION AND HYDROMETER METHOD.

should be the reverse, as one would think that some of the fines would undoubtedly go down with the larger particles, thus resulting in a coarser curve. Sometimes there is a small amount of sedimentation on the shoulder of the hydrometer weighing it down and causing a smaller hydrometer reading. This also would result in a coarser curve.

The application of a certain correction to the fundamental formula of Equation (3) has been proposed[11] by Mr. H. A. Lorentz, in order to take care of the increased force of friction due to frictional retardation of particles falling contiguous to the cylindrical glass wall. For hydrometer gradation purposes, however, this correction is entirely insignificant. Stokes' law gives the vertical movement of the particles; their diagonal motion, if any (especially in the lower portion of the gradation), will result in slightly higher values. Dr. Bouyoucos suggests a correction of —1 for hydrometer readings below 10 grammes per liter.

Both the elutriator and hydrometer methods are based on Stokes' law which considers particles as spherical in the derivation of the formula, and too many flat bodies in the sample will give unsatisfactory results in both cases.

The hydrometer method described herein will give reasonably accurate results, at least for the gradation of hydraulic-fill dam materials, for readily dispersible and granular soils. Undoubtedly, this method is very useful and of great assistance during periods when the results are necessary in the shortest time possible. Due to its many advantages it has become very popular and is being used in almost every soil analysis laboratory. However, the method of carrying out the test has not been standardized, as evidenced by discussions with other engineers and in the comparison of results obtained for the same soil sample in different laboratories. The tests at the Cobble Mountain Laboratory were made according to the theory and method of procedure outlined in this paper.

Turbidity

The standard turbidity test method can be used in estimating the percentage of a certain size of fine particles in a sample. This is only a rough approximation, and since the introduction of the aforementioned hydrometer method it has lost its value. If no hydrometer can be obtained, however, the turbidity method may be used to advantage.

Curves of particle diameter against distance below water surface are plotted from Equations (8) and (9) for various values of t. They indicate the minimum depth of various sizes of particles below the water (temperature, 10° cent.) surface after t min. of settling. In Fig. 7 the two horizontal curves give the turbidity of two different concentrations after various time intervals, t, during settling. On the right-hand side of the diagram are shown the corresponding readings of turbidity, in parts per million.

The turbidity readings will depend on various factors, such as: (1) The personal equation of the observer, especially the eyesight; (2) the intensity

[11] "Colloid Chemistry", by The Svedberg, 1928, p. 147.

of the daylight, or the lack of it; shadows on the sample; (3) the shape of the particle in suspension, whether spherical or flat; (4) the color of the sample; (5) presence of organic matter in the sample; (6) temperature effect; and (7) flocculation of the material. Even with these factors, however, for the same observer and under identical conditions, there will be, according to Fig. 7, a definite relation between percentage under a certain size of fine particles and the turbidity reading of the same material.

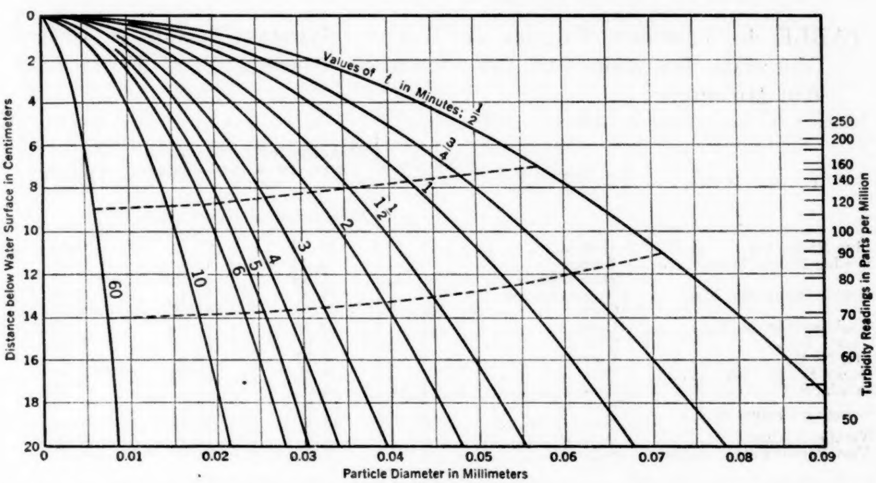

FIG. 7.—TURBIDITY CURVES SHOWING THE RELATION BETWEEN TURBIDITY AND STOKES' LAW.

Since the percentage finer than 0.01 mm. was used as a criterion, an attempt was made to find the relation between that percentage and the turbidity reading of the same material. The apparatus and the method of procedure are essentially the same as that used by the United States Geological Survey.[12]

The original concentration can be expressed in grammes of soil per liter of water. The turbidity reading is in parts per million and this is reduced to grammes per liter. This, then, divided by the original concentration, gives the percentage of original soil held in suspension at the time of observation. This ratio is called the percentage of turbidity.

A careful analysis of the same soil sample is made by a dependable method and the gradation is plotted. The percentage, from the gradation curve, finer than 0.01 mm. is divided by the percentage of turbidity, and the result is called the turbidity factor. When the percentage of turbidity and the turbidity factor of the same type of material are known, the product of the two will give the percentage of material finer than the particular size for which the factor was obtained.

Unless the material is completely broken up in the concentration while the turbidity is being taken, the turbidity reading will be very low. In some

[12] "Measurement of Color and Turbidity in Water", *Circular 8*, Div. of Hydrography, 1902.

of the tests made at Cobble Mountain Reservoir of foreign samples which flocculated, it was found that the actual turbidity percentage obtained after complete dispersion of the material in suspension was as high as four times the values obtained before deflocculation. (See Table 4.)

Readings were made 2 min. after the stirring had stopped. Various turbidity readings of the same sample were taken at intervals varying from 30 sec. to 1 hour after the stirring had stopped, and it was discovered that there

TABLE 4.—Turbidity Factors of Various Samples Tested at Cobble Mountain Reservoir for Percentage of Particle Sizes Finer than 0.01 Millimeter

Location of dam	Location of samples	Number of tests made	Percentage of Material Finer Than 0.01 Millimeter		Turbidity factors
			By elutriation	By turbidity	
Blue Ridge, Ga.	Core	1	23.0	36.1	0.64
Borden Brook, Mass.	Borrow	2	8.33	17.2	0.48
Cobble Mountain, Mass.	Stow borrow	60	1.89	4.16	0.45
	Russell borrow	21	11.3	18.3	0.62
	Core	103	10.56	21.25	0.50
Conklingville, N. Y.	Core	3	10.45	17.10	0.61
Garza, Tex.	Core	1	10.6	17.12	0.62
Rio, N. Y.	Borrow	1	4.51	6.8	0.65
Rocky River, Conn.	Borrow	2	9.46	17.45	0.54
Saluda, S. C.	Core	1	34.0	55.7	0.61
Swinging Bridge, N. Y.	Borrow	1	13.6	19.8	0.69
	Core	5	20.3	28.9	0.70
Westfield, Mass.	Borrow	2	10.41	17.4	0.60
West Parish Filters, Mass.	Borrow	1	7.37	12.97	0.57

was only a slight difference in the turbidity readings during the first hour of settling after the first 2 or 3 min. Evidently, the large particles, in the early stages of settling (which were above the platinum wire) had only a slight effect in reducing its visibility.

Organic Matter

Organic matter is present near the surface in almost all borrow-pits, and shallow pits will have a higher percentage than the deep ones. As a rule the surface soil of a dam site is sluiced, especially within the core area, and contains a high percentage of organic matter. The surface soil at the site of Cobble Mountain Dam varied from scarcely any to about 6 ft. in depth, and the average of sixteen samples showed about 8 per cent.

The determination of the percentage of organic matter is one of the important requirements. The organic matter that settles in the pool with the core material is usually very fine. It will tend to prevent the drainage of the core; it will affect the determination of the percentage of voids by moisture content and possibly the percolation test results; and it is often responsible for loss of material during elutriation tests. The effect of organic matter on computations of the percentage of voids (Equation (1)) may be expressed as follows:

$$V_i = 100 \left(1 - \frac{W}{B s_t}\right) \dots\dots\dots\dots\dots\dots\dots\dots (12)$$

and,

$$V_o = 100 - \left(\frac{W}{B}\right)\left(\frac{a}{s_o} + \frac{100 - a}{s_i}\right) \dots\dots\dots\dots\dots (13)$$

or, from Equations (12) and (13),

$$V_o \doteq V_i - a\left(1 - \frac{V_i}{100}\right)\left(\frac{s_i}{s_o} - 1\right) \dots\dots\dots\dots\dots (14)$$

in which, V_o is the percentage of voids observing effect of organic matter; V_i, percentage of voids neglecting effect of organic matter; s_o, specific gravity of organic matter; s_i, specific gravity of soil particles; a, percentage of organic matter by weight; W, weight of sample, in grammes; and B, volume occupied by sample, in cubic centimeters. The last term on the right-hand side of Equation (14) is a correction because of organic matter to be applied to the computations of percentage of voids, V_i, as ordinarily computed.

Assuming that $s_i = 2.72$, and $s_o = 0.60$, Equation (14) becomes:

$$V_o = V_i - 3.53\, a\left(1 - \frac{V_i}{100}\right) \dots\dots\dots\dots\dots\dots\dots (15)$$

It is interesting to study the relation between V_o and V_i for various percentages of organic matter, according to Equation (15). For instance, for a 3% organic matter ($a = 3$), with the computed percentage of voids, $V_i = 53$, it is seen that the actual percentage of voids is only 48. In other words, 3% of organic matter in the core material makes a difference of 5% in the computations of the percentage of voids of the material, had the organic matter been neglected. Equation (15) will apply for any other values of s_i and s_o and for their different values curves can be plotted to facilitate computation.

Organic matter can be determined by the method of "loss on ignition." This method is not reliable, unless the test is run carefully because minerals may decompose during the test if the sample is overheated, thus giving higher results for organic matter.

The albuminoid ammonia method was used for a few determinations of organic content, but was discarded later since reasonably accurate results could be obtained more easily and in a shorter time by igniting a sample.

Colorimetric tests for the determination of organic matter, similar to those made for organic impurities in concrete sands, were tried. Due to the fact that the cores of hydraulic-fill dams have a higher percentage of organic matter than concrete sands, a smaller ratio of sample material was used. It may be possible to shorten and simplify this method so that it can be used to advantage in the determination of organic content of materials for hydraulic-fill dams.

Gradation of Core Materials

With due respect to the difference in opinions regarding the many factors affecting the suitability of the core material other than its size gradation, the writer believes that one of the most important criterions to decide this suitability is the grain size and the gradation of the core material.

There has been much controversy among engineers in regard to the proper sizes of the core material for hydraulic-fill dams. Every engineer who has

built a successful dam is inclined to insist that the sizes of his core are the proper ones to be used. Unfortunately, every engineer has his own method of determining gradation, and the question arises as to whether the different results give a fair basis of comparison.

Numerous samples of core and borrow materials from important hydraulic-fill dams were secured and tested at the Cobble Mountain Reservoir with the same method and under the same conditions, so as to make the results comparable. In Fig. 8 the numbers in parentheses denote the number of

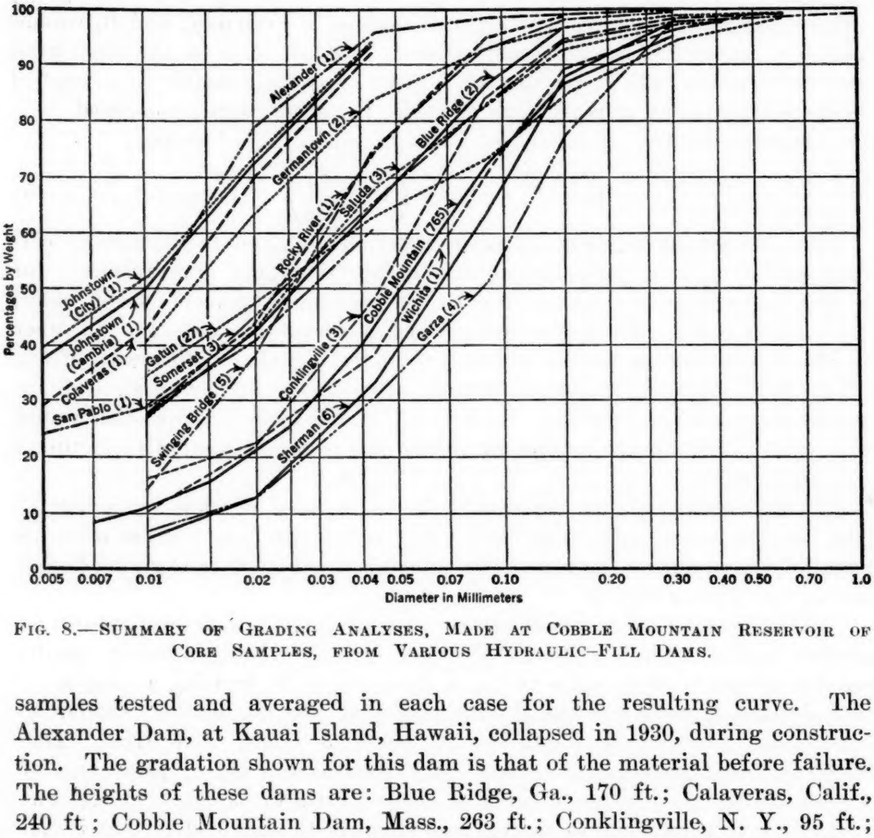

FIG. 8.—SUMMARY OF GRADING ANALYSES, MADE AT COBBLE MOUNTAIN RESERVOIR OF CORE SAMPLES, FROM VARIOUS HYDRAULIC-FILL DAMS.

samples tested and averaged in each case for the resulting curve. The Alexander Dam, at Kauai Island, Hawaii, collapsed in 1930, during construction. The gradation shown for this dam is that of the material before failure. The heights of these dams are: Blue Ridge, Ga., 170 ft.; Calaveras, Calif., 240 ft ; Cobble Mountain Dam, Mass., 263 ft.; Conklingville, N. Y., 95 ft.; Garza, Tex., 80 ft.; Gatun Dam, Canal Zone, 115 ft.; Germantown, Ohio, 110 ft.; Cambria Steel Company Dam, Johnstown, Pa., 95 ft.; City Dam, Johnstown, Pa., 70 ft.; Rocky River Dam, New Milford, Conn., 92 ft.; Saluda, S. C., 208 ft.; Sherman Dam, Monroe, Mass., 100 ft.; Somerset, Vt., 110 ft.; Swinging Bridge Dam, Forestburg, N. Y., 135 ft.; and Wichita Falls, Tex., 100 ft.

Scarcely, if ever, will the core material of a hydraulic-fill dam have the same grain sizes and gradation throughout the body of the core. One sample, or a few samples, of any core will not necessarily correspond to a represen-

tative average of the entire core material. It is necessary, therefore, to make frequent and numerous gradation analyses in order to obtain a fair average gradation curve for the entire core of a dam. The writer doubts whether the gradations given in the summary sheet (Fig. 8) will really represent the average of the entire core of any dam represented by the analysis of any one of its samples. Unfortunately, it was impossible to secure the desired number of samples for every dam. Otherwise, the summary sheet, Fig. 8, is believed to be of real value for a comparison between various core material gradations, since no error is present due to difference of method and to personal equations. It is well to call attention here to the fact that each engineer plots gradation curves differently. The gradation plotted in the first quadrant, with "diameter sizes, in millimeters" increasing from left to right as abscissas and "percentage of material finer by weight" increasing from the bottom upward as ordinates, seems to be the most logical.

Coefficient of Friction

The beach materials of hydraulic-fill dams consist generally of clean sand and gravel. The coefficient of friction of these materials is of great importance in design, but, unfortunately, no reliable data are available.

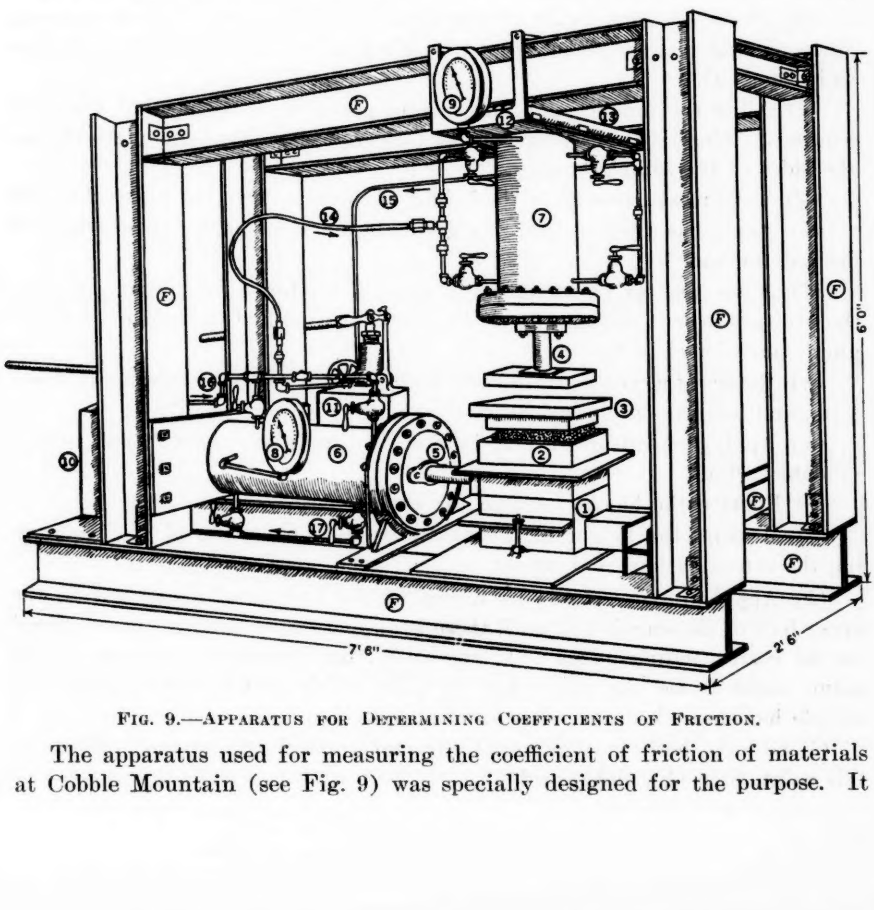

Fig. 9.—Apparatus for Determining Coefficients of Friction.

The apparatus used for measuring the coefficient of friction of materials at Cobble Mountain (see Fig. 9) was specially designed for the purpose. It

consists of a rigid steel frame, *F*, electrically welded throughout, weighing about 2 tons. Within this frame the essential working parts (labeled in Fig. 9) are, as follows: Vertical pressure is applied to the material within the sample boxes (1) and (2), by means of the loading block (3), which, in turn, gets its force from the piston shaft (4), operated by water under pressure within the cylinder (7). This hydraulic pressure is obtained by a hydraulic hand-pump (11), by way of the feed-pipe (14). The horizontal force required to shear the sample is applied to the upper box (2), in a similar manner by pump (10), feed-pipe (16), horizontal cylinder (6), and piston shaft (5). Hydraulic pressure in both the horizontal and vertical cylinders is recorded on 1 500-lb. pressure gauges (8) and (9), respectively. The vertical cylinder is mounted on a carriage (12), holding rollers (13), to permit horizontal movement as shear takes place. As the piston within the cylinders moves in either direction, water is forced back to the pumps through return pipes (15) and (17) for further use.

The procedure for determining coefficients of friction is:

(*a*) Weigh out about 100 lb. (dry weight) of sample and place enough of it in the lower sample box (1), agitating the material in the box under water for complete saturation.

(*b*) Place the upper half of the sample box (2) into position and continue placing the sample material to the top of the box in the same manner as in Step (*a*).

(*c*) Allow the sample material to drain partly for 5 to 10 min. Place the loading platform (3), leaving sufficient clearance between the platform and the sides of the sample box.

(*d*) Take measurements of the height of the sample in the box and record.

(*e*) Apply the vertical load (4) gradually on the sample, to pressure (9) desired for test.

(*f*) Allow the sample material to consolidate for 5 to 15 min., keeping the pressure over it constant by a slight pressure applied to the handle of the pump (mentioned in Step (*e*)).

(*g*) Release the vertical pressure, jack the two sample boxes apart about ¼ in., and leave them blocked apart.

(*h*) Apply vertical pressure as in Step (*e*) and allow the material to consolidate further.

(*i*) Remove the blocks between the sample boxes.

(*j*) Measure the height of the compressed sample material (for computing the consolidation) and record.

(*k*) Apply pressure slowly by means of the horizontal piston (5) to the upper half of the sample box until there is no increase in the pressure observed on the corresponding gauge (8); any additional horizontal pressure at this point would cause the upper box to slide which would indicate that the sample has sheared.

(*l*) Observe both the horizontal (8) and vertical (9) gauge readings at this point, Step (*k*), and record.

(*m*) From the foregoing records compute:

$$L = \frac{G A_p}{A_s} \qquad \text{(16)}$$

in which, L is the vertical pressure, in pounds per square inch; G, the vertical gauge reading; A_p, the area of the piston; and A_s, the area of the sample. Next compute the coefficient of friction from the relation: Coefficient of friction $= \frac{\text{horizontal gauge reading}}{\text{vertical gauge reading}}$. Finally, the percentage of voids is determined from,

$$V = \frac{100}{B}\left(B - \frac{W}{s}\right) \qquad \text{(17)}$$

in which, B is the measured volume of sample; s, the specific gravity of material; and W, the dry weight of sample.

(*n*) Release the pressures and remove the sample.

(*o*) Make independent and complete coefficient of friction tests, repeating Steps (*a*) to (*n*) for various desired vertical pressures up to 400 or 500 lb. per sq. in.

(*p*) Plot the results with the coefficient of friction as ordinate, and vertical pressure, in pounds per square inch, as abscissa.

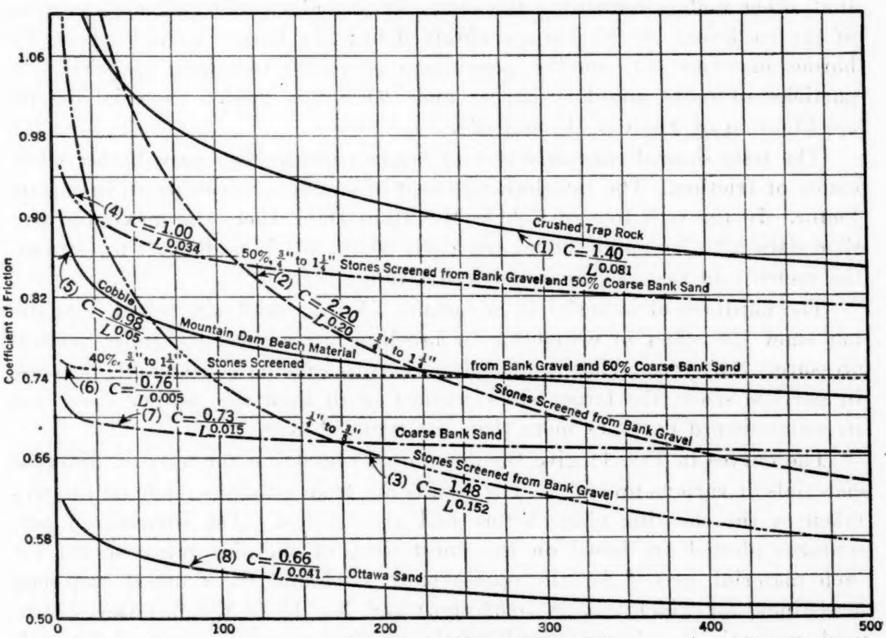

FIG. 10.—COEFFICIENT-OF-FRICTION CURVES FOR VARIOUS MATERIALS.

Curves resulting from about 550 coefficient-of-friction tests made at Cobble Mountain Reservoir, are given on Fig. 10. It will be seen that the coefficient is not constant, but is expressed by the general formula:

$$C = \frac{k}{L^m} \quad \text{.........................(18)}$$

in which, C is the coefficient of friction; k, the constant; L, the vertical pressure in pounds per square inch; and m, the exponent of L.

It is further seen that k, the constant for any one material, varies from 0.66 to 2.20, and that values of m, the exponent of L, range from 0.005 to 0.20, depending on the characteristics of the material tested.

Due to the fact that the beach materials of a hydraulic-fill dam are placed by water and that after construction they are never absolutely dry, the test materials were placed by water in the sample boxes, and, after draining all free water, the tests were conducted while the material was wet. The data given in Fig. 10 are based on the foregoing conditions.

Although primarily interested in the coefficient-of-friction values of the beach and core materials of Cobble Mountain Dam, more than 200 tests were made on outside materials consisting of various local and foreign samples of sand, gravel, crushed trap-rock and screenings, and Ottawa sand. The coefficients for the materials in question are affected by their particle sizes, hardness, percentage of voids, moisture content, particle shape, and gradation.

The conclusion drawn from the tests at Cobble Mountain Reservoir is that, other factors remaining the same, for any given material, higher values of the coefficient of friction are obtained by: (1) Larger particle sizes; (2) harder material; (3) smaller percentage of voids; (4) drier material; (5) particles of more angular shapes; and (6) better graded material, or the combination of Factors (1) and (3).

The tests showed conclusively that larger particle sizes gave higher coefficients of friction. The interlocking effect of stones seems to be an important factor. In the rock toes of Cobble Mountain Dam where the maximum particle sizes with irregular shapes are about 20 cu. yd., it is difficult to estimate the increase in the coefficient-of-friction value.

The hardness of material is important. No material was tested, that did not show the effect of crumbling or breaking after shearing under vertical pressure. Softer materials crumbled more, thus causing considerable decrease in particle sizes; the larger sizes crushed more than the smaller ones, and irregular-shaped particles more than the rounded ones.

The curves in Fig. 11 give the amount of crumbling for tests on different materials at various pressures. The results are from gradings made on samples taken at the shearing plane before and after a test. The increase in percentages plotted are based on the finest sieve of the size range shown for each material, except for the coarse bank sand and the crushed trap-rock screenings, for which No. 28 (0.62-mm.) and No. 14 (1.27-mm.) sieves were used, respectively. A new sample of the original material was used for each pressure. In every case, it is seen that the crushing effect increases with an increase in pressure, thus reducing the particle sizes. Undoubtedly, this is greatly, if not entirely, responsible for the decrease in the coefficient-of-friction values for higher pressures (Fig. 10).

For the same pressure a given material with a smaller percentage of voids yielded a higher value for the coefficient of friction. However, there is a limit to minimum percentage of voids because, under a given vertical pressure, the percentage of voids of a material will eventually become almost

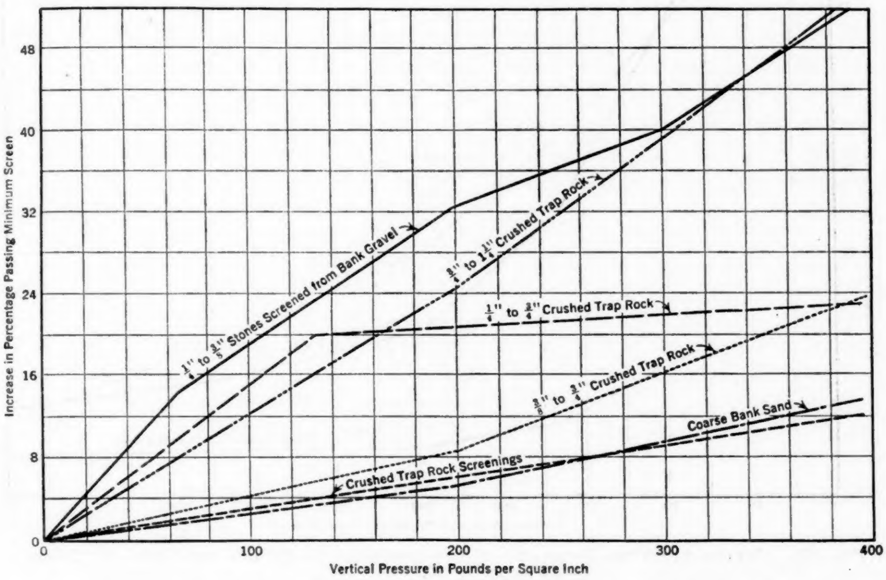

FIG. 11.—CURVES SHOWING AMOUNT THAT MATERIAL CRUSHED DURING TESTS FOR COEFFICIENT OF FRICTION.

constant. Slight variation in the consolidation under the same pressure may be due to the final arrangement or self-placement of the particles, or, perhaps, to the crumbling of particles to a certain extent.

When the particles of the material carrying the load are wet, the moisture seems to act as a lubricant, and there is a decided decrease in the value of the coefficients, especially at low pressure. The same material dried and tested for coefficient of friction under a given vertical load gives higher values. In Fig. 12, which illustrates this point, the increase in the coefficient of friction is based on the coefficient of the material in a wet condition; that is, the surfaces of all particles are entirely covered with water.

The more irregular the shape of particles the more pronounced is the effect of interlocking, resulting in increased resistance for friction. The easy crumbling of the sharp edges of particles with angular shapes may offset this interlocking advantage somewhat. Unless the material is hard, the sharp corners may not prove advantageous. The more uniform the particle sizes the higher is the percentage of voids. Well-graded beach material containing coarse particles has smaller percentages of voids and will have higher friction values.

The foregoing data are by no means all that are desirable as to the coefficient of friction of materials in hydraulic-fill dams. Unfortunately, at Cobble Mountain Reservoir it was not possible to make further investigations along

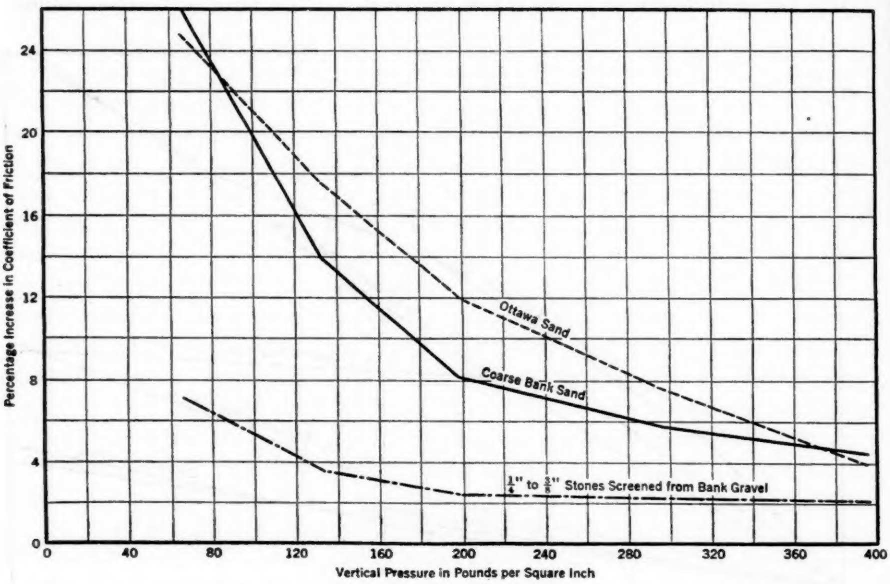

FIG. 12.—CURVES SHOWING INCREASE IN THE COEFFICIENT OF FRICTION DUE TO CHANGE FROM WET TO DRY CONDITION.

this line. In this large field much more reliable information is necessary to determine the characteristics of various materials, in order to understand fully the behavior of the coefficient of friction.

PERCOLATION

The percolation, or the permeability, test is one of the practical criterions of the ultimate behavior of the dam; and, therefore, this test is important and should be carried on with the greatest care.

The purpose of the gradation, or the determination, of the grain size of the core material is to ascertain whether the material has enough fines so that it will be impervious, or reasonably so, and whether it is coarse enough so that it will drain and solidify within the near future. As will be seen elsewhere in this paper, the effective size of the material, obtained from the gradation curve, will be useful in estimating its coefficient of permeability. It is better, however, to determine the coefficient of permeability directly rather than rely on gradation alone, as seepage through a core material will depend not only on the effective size and gradation of its materials, but also on its percentage of voids, its percentage of organic matter, its resistance to consolidation, or the shape and toughness of the particles, and on its percentage of clay, especially within the colloidal ranges, etc.

The velocity of seepage through fine core material is so slow that units, such as centimeters per second, or feet per second, will result in small quantities. It is advisable to express this velocity in meters per day, which will result in numbers that are more convenient. The coefficient of permeability, K, in meters per day, is also equal to $c d^2$, in which c is a constant and d is the effective size of the material.

As the normal ground-water temperature is about 10° cent., and as the change in temperature will affect the velocity, it is better to base the coefficient of permeability on 10° cent., or 50° Fahr. In dealing with the determination of the values of K, 45% voids consolidation was used as a basis. The fundamental formula used in determining the velocity of seepage through soils is known as Darcy's law:[13]

$$v = K \frac{H}{l_p} \qquad (19)$$

in which, v is the velocity of flow; K, the coefficient of permeability; H, the head causing the flow; and l_p, the length of the path of percolation. Since H and l_p are known, or can be assumed, the value of K is required. Due to the fact that the experiments to determine the value of K require long periods of time and constant watching, attempts have been made to develop short-cut methods.

Stand-Pipe Permeameter

Both the vertical capillary rise[14] and the horizontal capillary flow[15] methods of measuring permeability of soil were tried at Cobble Mountain Dam and were found unsuitable for core materials in a hydraulic-fill dam.

The simplest method of measuring the permeability of a soil is by a stand-pipe permeameter. In the apparatus used at Cobble Mountain shown in Fig. 13 the core material, in a saturated state similar to that in the dam, is placed in a glass tube and immersed in a pail of water. The head causing flow is measured from the top of the column of water in the stand-pipe to the level of the water in the pail. By means of a stop-watch the rate of fall of the water column in the stand-pipe was observed, and its volume was obtained when the bore of the stand-pipe had been determined. From the moisture content of the sample its percentage of voids is computed.

The coefficient of permeability, K, in meters per day, is computed from the equation:

$$K = 1\,989.44 \frac{A\, l_p}{A_s\,(T_2 - T_1)} \log \frac{H_1}{H_2} \qquad (20)$$

in which, A is the cross-section area of the stand-pipe; A_s, the cross-section area of the sample; l_p, the thickness of the sample; T, time; and H, the head that causes flow; that is, H equals height of water in the stand-pipe, H_1 at

[13] "Recherchs Expérimentales Relatives au Mouvement de l'Eau dans les Tuyaux," par Henry Darcy, Paris, 1857.

[14] "Soil Studies for the Granville Dam in Westfield, Mass.", by Charles Terzaghi, M. Am. Soc. C. E., *Journal*, New England Water Works Assoc., June, 1929, Vol. 43, No. 2, p. 191.

[15] "Soil Mechanics Research" by Glennon Gilboy, Jun. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., October, 1931, p. 1173, Equation (5).

the beginning, and H_2, at the end, of the run. Equation (20) is derived from Darcy's law and is reduced from the observed temperature to its value at 10° cent. by means of Fig. 5. During any of the tests—for the same material, with the same percentage of voids, and whatever the value of H_1 in

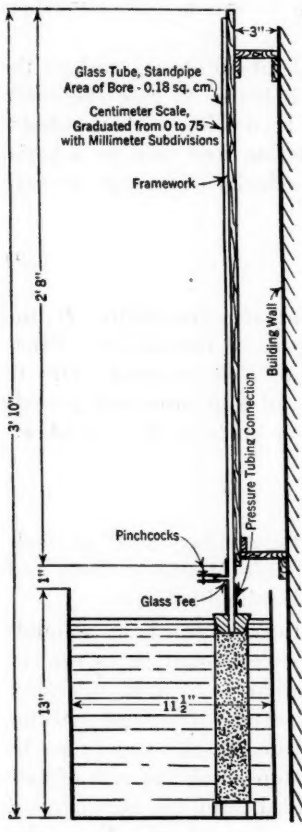

FIG. 13.—STAND-PIPE PERMEAMETER.

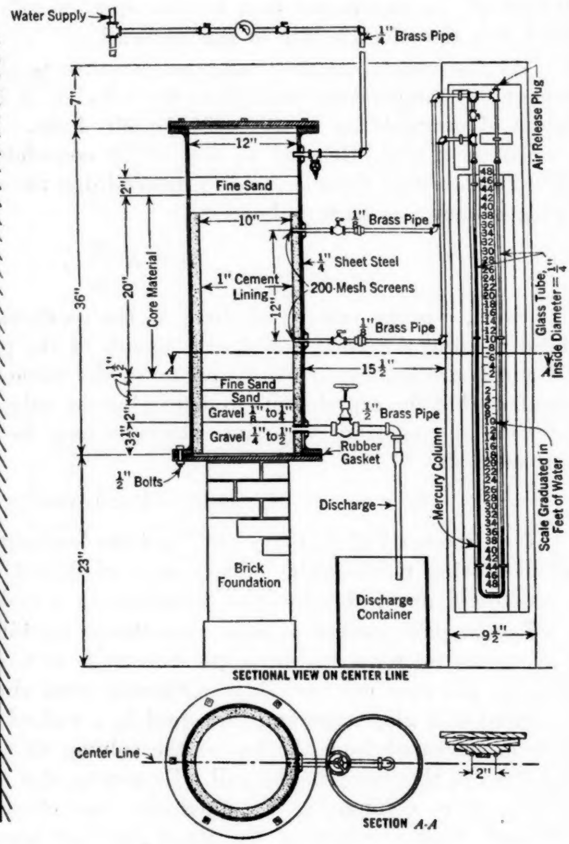

FIG. 14.—LARGE PERCOLATOR.

Equation (20)—K remained constant, provided the sample maintained structural equilibrium during the runs.

It should be remembered that the stand-pipe permeameter has a low head and the sample to be tested is very small. Furthermore, there is a chance for the water to flow more freely contiguous to the inside wall of the glass cylinder.

LARGE PERCOLATOR

This apparatus (Fig. 14), designed by the late Allen Hazen, M. Am. Soc. C. E., can be used with a high head of water pressure. It has a capacity for core material of about 1 cu. ft., which is sufficient to reduce the effects of

slight local variations in the sample. It was lined with concrete forming a collar near the top of the sample to check seepage contiguous to the inside surface (see Fig. 14).

The sample is placed in a saturated state as it comes from the core, and is supported and covered by sand filters. After a period of initial settlement (about 24 hours), the container is sealed and the water pressure is applied at the top of the sample in gradually increasing amounts, to the desired limit. A maximum head of water of about 60 ft., was used at Cobble Mountain Reservoir.

In order to avoid the effect of a possible film of fines at the surface of the sample, the head was measured by a differential mercury manometer connected at points, 1 ft. apart, lying within the sample. The readings gave a direct measurement of the head causing the flow through 1 ft. of the material, or the value of $\frac{H}{p}$ in Equation (19). The discharge was measured at convenient intervals, when the temperature was observed.

The values of K in Equation (19) decreased rapidly during the first few hours of the test, and continued to decrease slowly for a long time. The initial tests were carried on over a period of from two to three months, when the coefficient of permeability became nearly constant. It was found, however, that satisfactory comparable tests could be terminated in a week. The initial rapid decrease was due to the fact that the applied pressure produced a decrease in the percentage of voids which was accompanied by settling of the sample. All values of K were reduced to 10° cent. for comparison. The voids of the core sample were found at the beginning and at the end of the test, by moisture-content determinations.

The large percolator is satisfactory for the determination of the coefficient of permeability for a soil sample at a single percentage of voids, but it is not suitable for the measurement of consolidation, or the corresponding variations in the coefficient of permeability.

Standard Compression Apparatus

The best device for the determination of the relation between K and the percentage of voids, is that by Professor Terzaghi, described by Professor Gilboy in the paper previously cited. This device (called the standard compression apparatus) will measure not only the rate of consolidation and compressibility of a soil sample under a given load, but also its permeability at different stages of consolidation. No description of the apparatus is necessary here, as it has been described often and is fairly well known. The loading system is made in such a way that the compressibility of core material can be measured with a load of 200 lb. per sq. in., which is about the maximum pressure in the Cobble Mountain Dam.

After the sample is placed in the machine, the stop-pin is removed and consolidation is allowed to take place under the weight of the piston. When this is practically complete, the stand-pipe connection is made. Increments of weight are applied in order to give the desired pressures up to 200 lb.

per sq. in. At each loading observations are made to determine the rate of consolidation under that load and sufficient time is allowed for complete consolidation. The permeability after complete consolidation is observed for each loading.

The increments of weights are removed in the order that they were placed, and sufficient time is allowed for complete expansion between the removal of each load. Additional observations may be made for permeability at this time.

The average of the readings of two Ames dials is used for computing the consolidation. The coefficient of permeability is computed from Equation (20) and reduced to its equivalent value at 10° cent. The following data are obtained for a given material: (*a*) The rate of consolidation for each loading; (*b*) the compressibility and expansion; and (*c*) the coefficient of permeability.

Numerous tests were run with the standard compression unit and valuable data obtained. It was unfortunate, however, to note that the coefficient of permeability of the same sample and for the same percentage of voids obtained by the standard compression apparatus was invariably lower than, and in some cases one-half, the results obtained from the large percolator. The variation of the results, for each apparatus and for different percentages of voids, was consistent. There was no apparent reason to doubt the results obtained by the large percolator because no difference was found in the value of K for different values of H. The sample of the standard compression apparatus is so small that a personal equation or any slight cause may result in a great difference in the result of the test. The following might account for the small values of K obtained by the standard compression apparatus: (*a*) The inside area of the piston bearing on the top porous plate is 28.7 sq. cm., whereas the area of the sample is 38.1 sq. cm.; thus, the water of about 25% of the sample area is forced to come in a diagonal direction; (*b*) there is a possibility that the top porous plate may become clogged to a certain extent; and, (*c*) the puddling of the sample in placing it in the cylinder may cause segregation.

The results obtained for the coefficient of permeability of fine core materials are so small and subject to so many different factors that even, say, about 100% variation between two different apparatus cannot be considered serious.

Percolation and Percentage of Voids

In the numerous experiments for the coefficient of permeability of any one material, K remained constant for any value of H, provided the percentage of voids of the material remained constant and the sample maintained structural equilibrium, but it showed a decided change due to variation in the percentage of voids.

The curves of Fig. 15 show the relation between the value of K and the percentage of voids of several core samples. This relation can be expressed by the formula,

$$K = c\,p^m \qquad (21)$$

in which, K is the coefficient of permeability, in meters per day, and for $t = 10°$ cent.; c, is a constant; and p, the percentage of voids divided by 100. The value of c varied from 3.395 to 0.301 and the value of m, from 5.36 to 7.06.

For the same material, it can be stated that K varies approximately as the sixth power of the percentage of voids. Thus, the value of K for 50% voids compared with that of 40% will be about $\left(\frac{50}{40}\right)^6$, or 3.8 times. This relation, derived from tests on local core samples, will be reasonably accurate

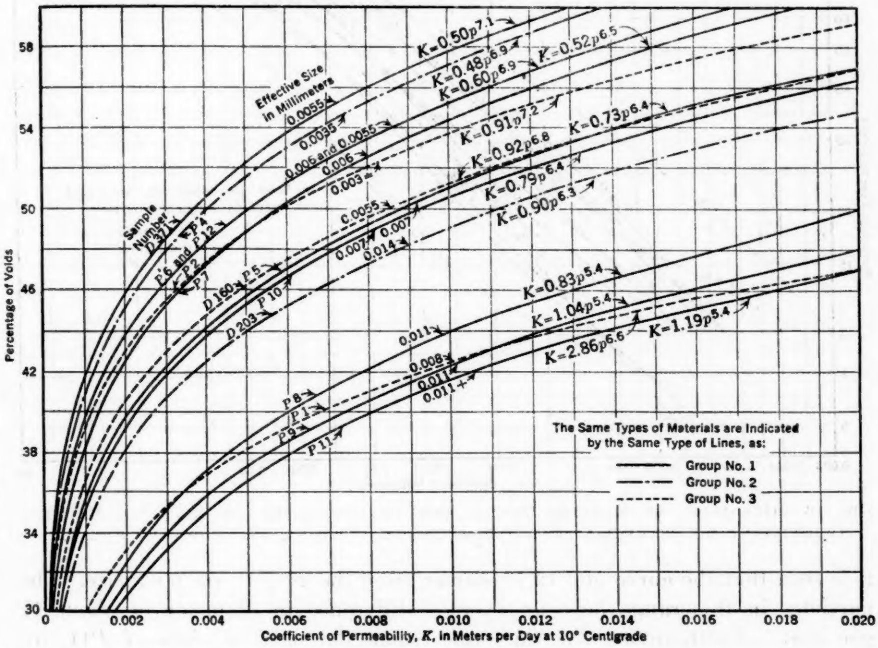

FIG. 15.—PERCENTAGE OF VOIDS PLOTTED AGAINST COEFFICIENTS OF PERMEABILITY FOR VARIOUS MATERIALS AND EFFECTIVE SIZES.

for local material. The relation might be similar for other core materials, provided their grading and character are similar.

In Fig. 15, the values of K point out three distinct characteristics; namely, that they (1) differ considerably for different materials; (2) vary most decidedly according to the effective size of each sample of the same character of material; and (3) vary according to the slope of the gradation curves of the same character of material even if the materials may have the same effective size.

Fig. 16 gives the gradations of the materials of the curves of Fig. 15. Comparing $P5$ and $P10$, both being the same kind of material and having the same effective size—0.007 mm.—and the value of m being 6.4 in both cases,

the value of c for $P5$ is 0.73, and for $P10$, it is 0.79. On Fig. 16, it is seen that $P5$ is finer than $P10$ with a difference of about 0.005 mm. in their grain sizes for the entire range between the 35% size and the 60% size, or the $P10$ grain sizes are 16% larger than $P5$ in that range. This causes about 8% variation in the rate of percolation of the two samples.

A similar comparison of $D371$ with $P12$ shows that the latter has a coefficient of permeability about 40% greater than the former. From Fig. 16

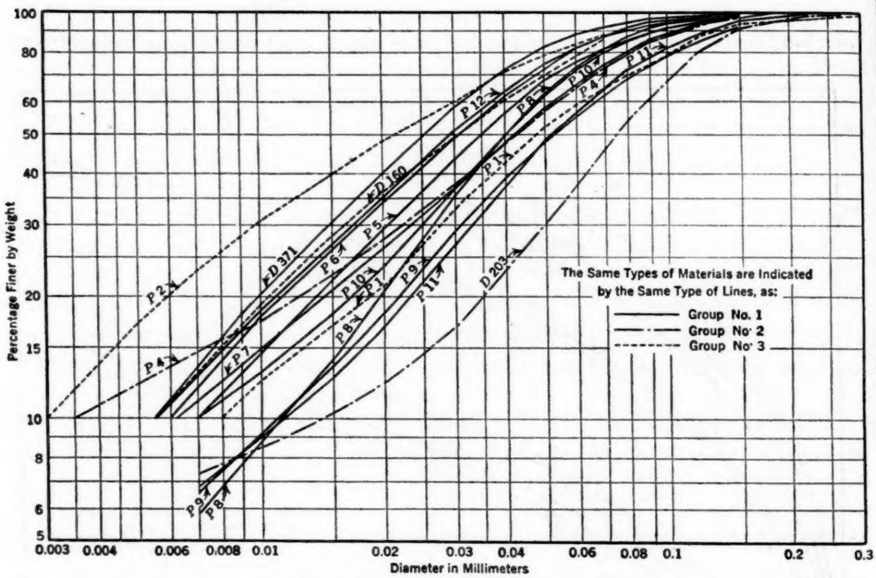

FIG. 16.—GRADATION OF SAMPLES TESTED FOR DETERMINATION OF RELATION BETWEEN COEFFICIENT OF PERMEABILITY AND PERCENTAGE OF VOIDS, AS SHOWN IN FIG. 15.

it is seen that the curve of $P12$ is coarser from the 20% to the 60% size. The particles in this range have an average difference in diameter of about 17 per cent. Furthermore, for the same range the particle sizes of $P11$ are about 27% greater than those of $P8$, while $P11$ is about 43% more permeable than $P8$. This, in spite of the fact that the effective sizes of the two samples, in both the foregoing cases are the same.

EFFECTIVE SIZE

Many claim that effective size is an arbitrary term, stating that it may suffice for filter sand where the gradation is more or less uniform and doubt its application to a core material. It is true that without the uniformity coefficient the effective size of a material will give no idea in regard to its gradation. A great many curves can have the same effective size with enormous difference in the gradation of the materials. It should be remembered, however, that the minimum and maximum 60% sizes of the core materials (Fig. 8) are about 0.013 mm. and 0.11 mm., respectively, and that their average value is about 0.035 mm. The effective size term, when applied

to a core material, therefore, gives a fair notion of the gradation of that material, especially when the term definitely fixes in mind the maximum size of the 10% fines in the total core.

Mr. Hazen's formula[16] for filter sands is:

$$v = c\,d^2 \frac{H}{l_p}\left(\frac{t+10}{60}\right) \qquad (22)$$

It is taken from Darcy's law as expressed by Equation (19), in which, v is the velocity of water, in meters per day; c, a coefficient; d, the effective size, in millimeters; H, the head causing the flow; l_p, the depth of material (filter sand in Hazen's formula); t, the temperature, in degrees Fahrenheit; and K, the coefficient of permeability.

For $t = 50°$ Fahr., or $10°$ cent. (which is the normal temperature for ground-water) Hazen's $c\,d^2$ becomes equal to Darcy's K. Numerous tests were run to determine the coefficient of permeability, K, for the core material of Cobble Mountain Dam, and, at the same time, to find out whether $c\,d^2$ would be applicable to flows through the extremely fine core materials. The effective sizes of these materials subject to experiments varied from 0.003 mm. to 0.012 mm., and it was discovered that at 45% voids the value of c remained reasonably constant, with a mean average of about 150. The conclusion was reached that Hazen's factor, $c\,d^2$, can be applied approximately for seepage through these fine core materials with $c = 150$. It should be remembered that in using $K = c\,d^2 = 150\,d^2$, K is the coefficient of permeability for velocity, in meters per day, for $t = 50°$ Fahr., and at 45% voids. For any other temperature, the factor, $\frac{t+10}{60}$, of Equation (22) should be applied and for different percentages of voids Equation (21) can be applied; or perhaps it can be better expressed by:

$$K = 150\,d^2\left(\frac{V}{45}\right)^6\left(\frac{t+10}{60}\right) \qquad (23)$$

in which, K is the coefficient of permeability, in meters per day; d, effective size, in millimeters; V, percentage of voids; and t, temperature, in degrees Fahrenheit.

The value of c may vary considerably from 150, due to variations in the character and gradation of different core materials. For a given locality, however, where the materials are of a similar character and grading, c can be evaluated and used with reasonable accuracy.

It is better, of course, to conduct tests to measure directly the coefficient of permeability of the core material of a dam under construction, in order to calculate, more or less accurately, the probable seepage through the dam, but Equation (23) is a handy tool for computing at least approximately, the probable seepage through the dam before its construction. This might result in very effective changes in the design of the contemplated structure, or in locating other than the anticipated borrow-pits to be used for the construction.

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[16] Annual Rept., Massachusetts State Board of Health, 1892, p. 539.

The significance of effective size is appreciated and its importance is realized due to the fact that the seepage, all other factors influencing it remaining the same, varies as the square of the effective size. Thus, a computed seepage through a dam in which the material has an effective size of 0.008 mm. will be increased nine times if the effective size is changed to 0.024 mm.

It is hoped that in the future, with more comprehensive investigations, a factor may be obtained for Equation (23) to take into consideration the effect of the slope of the gradation curve. (See Characteristic (3) under "Percolation and Percentage of Voids.")

Effective Size Estimated by Observation

It is surprising to know how easily an inspector at the dam can be trained to judge the approximate effective size of the core material by visual examination and touch. The method of comparing the material from the dam with a sample of similar material that has been tested, is used to train the inspector, and to maintain a constant field check on the work. A series of samples having effective sizes from 0.002 to 0.02 mm. were in the field for this purpose.

It was discovered that a core material with an effective size of about 0.002 mm., when rubbed between the fingers, felt like graphite; it was greasy like butter. An experienced inspector can tell the character of a core material in regard to its fineness by observing the flow while it is going over the beach. These observations may not apply directly to other dams, but similar observations may be found useful.

Consolidation of the Core of Cobble Mountain Dam

An attempt has been made to estimate roughly the time necessary for an approximate ultimate consolidation of the core at various heights of the Cobble Mountain Dam. By "approximate consolidation" for various heights of the dam is meant the state of consolidation that the material at any height will attain due to the pressure or weight of the structure itself above that point.

In the laboratory, numerous samples of core material were tested in the standard compression apparatus and corresponding consolidation curves were obtained. For a given material, these curves showed the ultimate percentage of voids of the material under a given load. The application of pressure was made in increments from a very small load up to 200 lb. per sq. in.

It was discovered that the ultimate percentage of voids was affected by the arrangement of the particles (the percentage of voids) at the beginning of the tests and by the effective size of the material. The lower the percentage of voids at the beginning of a test, the lower will be the final percentage of voids. With materials of similar gradation, the larger the effective size, the lower will be the percentage of voids throughout the test.

The average of the laboratory results for material with an effective size of 0.008 mm., can be expressed by the equation:

$$100 - V = 49.3\, L^{0.0428} \quad \text{(24)}$$

in which, V is the final percentage of voids (expressed in whole numbers, thus, $45\% = 45$ and not 0.45); and L is the pressure, in pounds per square inch, causing ultimate consolidation to $V\%$ voids.

It is questionable whether the consolidation formula obtained in the laboratory will apply to the actual consolidation of the same material in the construction of the dam because of the following reasons:

1.—The applications of the loads in the laboratory and in the dam are different.

2.—The initial percentage of voids or the arrangement of the particles in the testing machine and in the dam is not the same.

3.—The sample for the test in the standard compression machine is very small. It was found that there is a great difference in the time necessary for the ultimate consolidation during a given period under a given loading.

4.—There is a great difference between the inside polished surface of the standard compression unit of the apparatus and the rough surfaces of the abutments and the beaches in the dam.

5.—The time interval for a given load is different.

6.—The drainage facilities are not the same.

7.—There is the question of the load per square inch at any one given depth in the dam, due to the uncertainty of the behavior of the core material in coming to a state of structural equilibrium, or in the rate of consolidation taking place in the upper part of the dam above that particular point. As the percentage of voids in the core decreases, the weight of the core material per cubic foot increases, thus causing a higher pressure per unit area at any depth of the dam.

The consolidation data obtained for Birch Meadow borrow materials may not necessarily apply to other borrow materials in the dam. The results and the formula obtained, should be used with discretion. The application of the formula may not be entirely correct with respect to the actual conditions in the dam; but at present it is the best available information, and, undoubtedly, it is a tool that can be utilized to compute approximately the time of consolidation at various depths.

At the completion of the dam, if the percentage of voids of the core material at any height can be assumed, and the final percentage of voids ascertained from the consolidation formula at that height, the quantity of water to go through a vertical area at the same elevation in the dam can be calculated. The width of the core at any height will determine the path of percolation of the water to be squeezed out at that height; hence, the total quantity of water that will have to drain away can also be determined. Using a certain average velocity of the water, the time necessary for it to drain away can be computed.

It is evident that, in addition to the uncertainties to the application of the consolidation formula, in order to solve the problem, other assumptions are being made, which perhaps are not exactly right.

For example, let B equal the volume of water, in cubic meters, to drain from the core material with a given percentage of voids until it attains the

final percentage of voids under a given load; Q, the discharge, in cubic meters per day; v, the average velocity, in meters per day, for V and V'; A, the vertical area, in square meters, perpendicular to direction of flow; T, the number of days; H, the height, in meters, or in feet, below the top of the dam; D, the width of core, in meters, or in feet, at H distance below the top of the dam; V, the initial percentage of voids of core material; V', the final percentage of voids of core material; and d, the effective size of core material, in millimeters.

Assume that horizontal and longitudinal drainage of the water occurs in the core material during consolidation. (This is not exactly the case, as there may always be vertical and diagonal drainage.) Assume, also, that the reservoir is full and that the water in the core material during consolidation cannot flow up stream and that it will drain the full width of the core horizontally down stream. Then, $B = QT = AvT$, or,

$$T = \frac{B}{Av} \qquad \text{.........................(25)}$$

The volume of water, B, to be drained within a section of the core during the change of percentage of voids from V to V' is equal to the original minus the final water content of the section.

With a percentage of voids of V, there are $\frac{V}{100} DA$ units of water and $\frac{100 - V}{100} DA$ units of solid material. When the percentage of voids becomes V', there are $\frac{V'}{100} DA'$ units of water and $\frac{100 - V'}{100} DA'$ units of material (in which, A' is the reduced area due to consideration); but, $\frac{100 - V}{100} DA = \frac{100 - V'}{100} DA'$, or,

$$A' = A \, \frac{100 - V}{100 - V'} \qquad \text{.........................(26)}$$

and $B = \frac{V}{100} DA - \frac{V'}{100} DA'$, or,

$$B = \frac{D}{100} (AV - A'V') \qquad \text{....................(27)}$$

Substituting the value of A' from Equation (21) in Equation (27),

$$B = DA \, \frac{V - V'}{100 - V'} \qquad \text{....................(28)}$$

The rate of consolidation of the material under a given constant load, shortly after the application of the load, becomes almost uniform. It is reasonable to assume, therefore, that the average velocity of water drained during the consolidation from a percentage of voids of V to V' will be about the velocity at $\frac{V + V'}{2}$; then, for $A = 1$ sq. m.,

$$v = 150 \, d^2 \, \frac{H}{D} \left(\frac{V + V'}{2 \times 45} \right)^6 \qquad \text{....................(29)}$$

Substituting Equations (28) and (29) (for $A = 1$) in Equation (25) and reducing the meter units to feet,

$$T = \frac{D^2\,(V - V')}{492\,d^2\,H\,(100 - V')\left(\dfrac{V + V'}{90}\right)^6} \quad \ldots\ldots\ldots\ldots\ldots (30)$$

It is evident from the time formula, Equation (30), that the lower the point under consideration in the Cobble Mountain Dam the longer it will take for final consolidation. Then the bottom of the dam only should be considered, say, Elevation 730.

Assuming that the average percentage of voids of the core material of the entire dam at its completion is 47.5 and that the specific gravity of the solid material is 2.72, the combined weight of water and material per cubic foot is 119 lb., corresponding to a specific gravity of 1.9, or 0.825-lb. pressure per sq. in. for each foot of head. The elevation at the top of the dam is 972; then at Elevation 730 the load will be $0.825 \times 242 =$ about 200 lb. per sq. in. The ultimate percentage of voids of the core material subject to a 200-lb. load per sq. in., is 38.15 according to Equation (24). Hence, V equals 45% at the bottom of the dam; V', 38.15%; D, 170; d, 0.008 mm.; and H, 222 (flow-line elevation = 952). Inserting the foregoing in Equation (30), $T = 735$ days.

Considering that—(*a*) there is always vertical drainage during construction; (*b*) after construction the drainage is not horizontal only; (*c*) it will take about a year for the reservoir to fill; and (*d*) during this period, if not also afterward, the drainage will take place horizontally both up stream and down stream, thus cutting in half the length of the path of percolation—it may be permissible to reduce the period of 735 days from 25 to 50%; in other words, the time necessary for the consolidation of the Cobble Mountain core may be from about 1 year to 18 months.

It should be remembered that Equation (24) is obtained with samples of an effective size of 0.008 mm. from the middle of the core. Naturally, the average effective size of the core is more than 0.008 mm., in which case the time necessary for the ultimate consolidation would be less.

As noted in Equation (30), T, number of days for consolidation, varies inversely as the square of the effective size. The average effective size for the entire core is 0.009 mm. Therefore, with an effective size of 0.009 mm. the number of days for consolidation will be, $T = 735\left(\dfrac{0.008}{0.009}\right)^2 = 580$ days. This is practically within the range of the aforementioned estimated time of 1 year to 18 months.

Seepage Formula for Hydraulic-Fill Dams

Now that an average approximate value has been ascertained for the coefficient of permeability of the core material, the problem is to apply this information so as to be able to calculate the seepage through a hydraulic-fill dam. As far as the writer knows, there is no such general formula giving the seepage through any hydraulic-fill dam; and the derivation of such a general formula is given in the following pages.

It is proposed to use Darcy's law (Equation (19)), in deriving such a general formula. The controlling factor in the seepage through any hydraulic-fill dam, with a known or assumed coefficient of permeability of its core material, will be the section of the core of that dam, provided the core has assumed a sufficient state of consolidation to make it stable. The simplest way that Darcy's formula can be applied is to assume a horizontal flow for the seepage, although it is a fact that such is not exactly the case, at least for the upper section of the core. The diagonal flow of the seepage through the core in the upper part of the dam may result in the reduction of the core area affecting the percolation. On the other hand, there may be some clogging of the upper surface of the core resulting in decrease of percolation. However, the general seepage formula in this paper will be accurate enough for all practical purposes in estimating approximately the loss of water by seepage through the core. Fig. 17 shows the maximum cross-section of Cobble Mountain Dam.

In Fig. 18, let D = width of core = path of percolation, l_p, at H-distance below the flow line; D_1 equal the width of core = l_p, at flow-line elevation; D_2, width of core = l_p, at bottom of the dam; l, length of dam at H-distance below the flow line; l_1, length of dam at flow-line elevation; l_2, length of dam at bottom; and, H_2, height of dam to flow-line elevation.

In the hypothetical case of a rectangular core section (see Fig. 18(a): $D = l_p = D_1 = D_2$ and $l = \dfrac{l_1 H_2 - (l_1 - l_2) H}{H_2}$. From Fig. 18($a$), $d A = l\, d H$, and the derivative of Equation (19) is,

$$d Q = K \frac{H}{D} d A = K \frac{H}{D} l\, d H \qquad (31)$$

Substituting values of D and l in Equation (31),

$$d Q = K \frac{H}{D_2}\left[\frac{l_1 H_2 - (l_1 - l_2) H}{H_2}\right] d H$$

or,

$$Q = K \frac{l_1}{D_2} \int H\, d H - K \frac{(l_1 - l_2)}{D_2 H_2} \int H^2\, d H + k$$

Integrating between the limits, $H = H_2$ and $H = 0$, and simplifying,

$$Q = K \frac{H_2^2}{6 D_2} (l_1 + 2 l_2) \qquad (32)$$

Equation (32) expresses the seepage for a length, l. In the general case of a trapezoidal core section, for any hydraulic-fill dam (Fig. 18 (b)),

$$D = \frac{D_1 H_2 + (D_2 - D_1) H}{H_2}$$

and,

$$l = \frac{l_1 H_2 - (l_1 - l_2) H}{H_2}$$

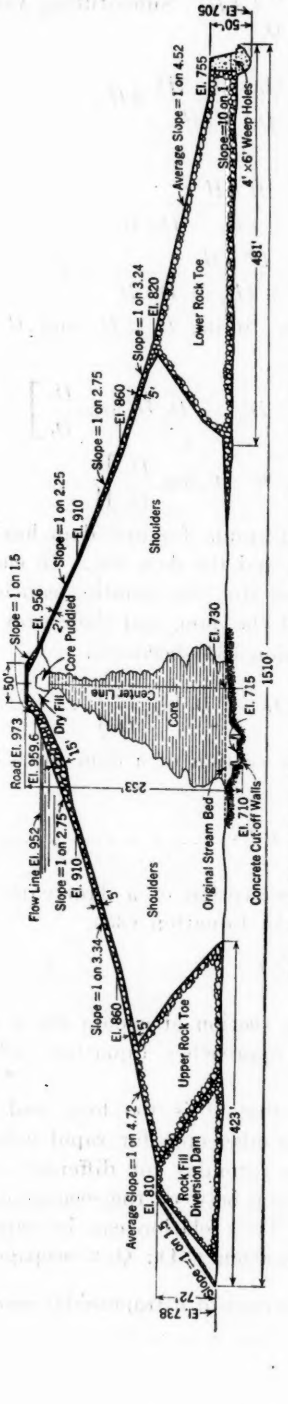

FIG. 17.—MAXIMUM CROSS-SECTION OF COBBLE MOUNTAIN DAM.

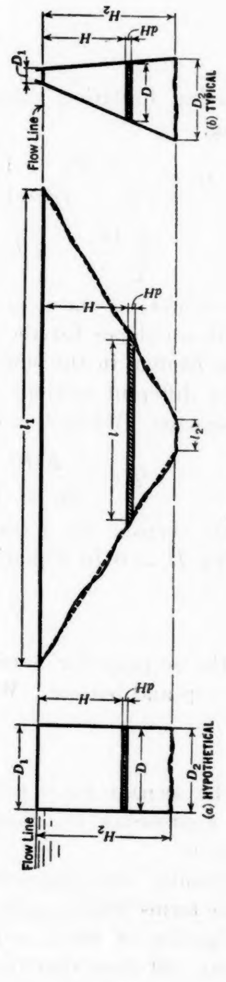

FIG. 18.—CORE SECTION.

As in the hypothetical case, $d\,Q = K\,\frac{H}{D}\,l\,d\,H$. Substituting values of D and l, as before,

$$d\,Q = \frac{K\,H\,H_2\,[l_1\,H_2 - (l_1 - l_2)\,H]}{[D_1\,H_2 + (D_2 - D_1)\,H]\,H_2}\,d\,H$$

or,

$$Q = K\,l_1\,H_2 \int \frac{H\,dH}{D_1\,H_2 + (D_2 - D_1)\,H} - K\,(l_1 - l_2) \int \frac{H^2\,dH}{D_1\,H_2 + (D_2 - D_1)\,H} + k \quad \text{.........(33)}$$

Integrating Equation (33) between the limits, $H = H_2$ and $H = 0$, and simplifying,

$$Q = \frac{K\,H^2_2}{(D_2 - D_1)^3} \left\{ l_1 \left[\tfrac{1}{2}\,(D^2_2 - D^2_1) - D_2\,D_1 \log_e \frac{D_2}{D_1} \right] + l_2 \left[\frac{D^2_2}{2} - 2\,D_2\,D_1 + \frac{3}{2}\,D^2_1 + D^2_1 \log_e \frac{D_2}{D_1} \right] \right\} \quad \text{............(34)}$$

Equation (34) is the general seepage formula for any dam having more or less uniform slopes for the core section and the dam site. In case of any appreciable change in the slopes of the dam site, the equation can be applied in steps for different sections or heights of the dam, and their sum will give the total seepage. When $l_1 = l_2 = l$, Equation (34) becomes:

$$Q = \frac{K\,H^2_2\,l}{(D_2 - D_1)^2} \left\{ D_2 - D_1 - D_1 \log_e \frac{D_2}{D_1} \right\} \quad \text{...........(35)}$$

which is the seepage for a trapezoidal core section of a dam for a length, l. Substituting $D_1 = 0$ in Equation (34),

$$Q = \frac{K\,H^2_2}{2\,D_2}\,(l_1 + l_2) \quad \text{.....................(36)}$$

which is the seepage for a triangular core section of a dam with different length for top and bottom. When $D_1 = 0$ in Equation (35),

$$Q = \frac{K\,H^2_2}{D_2}\,l \quad \text{........................(37)}$$

which is the seepage for a triangular core section of a dam for a length, l. When D_1 approaches D_2, Equation (34) approaches Equation (32), as its limiting value.

The difficulty with Equation (34) is that it is too long and contains logarithmic terms which make it rather cumbersome for rapid calculations. An investigation of these several seepage formulas for different geometric core sections will show that there is a relation between the seepage quantities and the areas of different core sections. This relation can be expressed as follows; let Q_1 = seepage according to Equation (34); Q_2 = seepage according to Equation (32); $A_1 = \frac{H_2}{2}\,(D_1 + D_2)$ = area of a trapezoidal core section,

Equation (34); and $A_2 = D_2 H_2$ = area of a rectangular core section, Equation (32). Then,

$$\frac{Q_1}{Q_2} = \frac{A_1}{A_2} X \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

in which, X is a factor depending on ratios of $\frac{D_1}{D_2}$ and $\frac{l_2}{l_1}$.

Substituting values of Q_1, Q_2, A_1, and A_2, and solving for X,

$$X = \frac{12\, D^2_2\, l_1 \left[\frac{1}{2}\,(D^2_2 - D^2_1) - D_2\, D_1 \log_e \frac{D_2}{D_1} \right]}{(D_2 + D_1)\,(D_2 - D_1)^3\,(l_1 + 2\, l_2)}$$
$$+ \frac{12\, D^2_2\, l_2 \left[\frac{D^2_2}{2} - 2\, D_2\, D_1 + \frac{3}{2}\, D^2_1 + D^2_1 \log_e \frac{D_2}{D_1} \right]}{(D_2 + D_1)\,(D_2 - D_1)^3\,(l_1 + 2\, l_2)} \ldots\ldots\ldots (39)$$

Substituting values of Q_2, A_1, and A_2, in Equation (38) and solving for Q_1,

$$Q_1 = \frac{X\, K\, H^2_2}{12\, D^2_2}\,(l_1 + 2\, l_2)\,(D_1 + D_2) \ldots\ldots\ldots\ldots\ldots (40)$$

but Q_1 is the seepage in Equation (34); therefore, the general equation for any trapezoidal core section for a hydraulic-fill dam becomes,

$$Q = X\, K\, H^2_2\,(l_1 + 2\, l_2)\,\frac{D_1 + D_2}{12\, D^2_2} \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

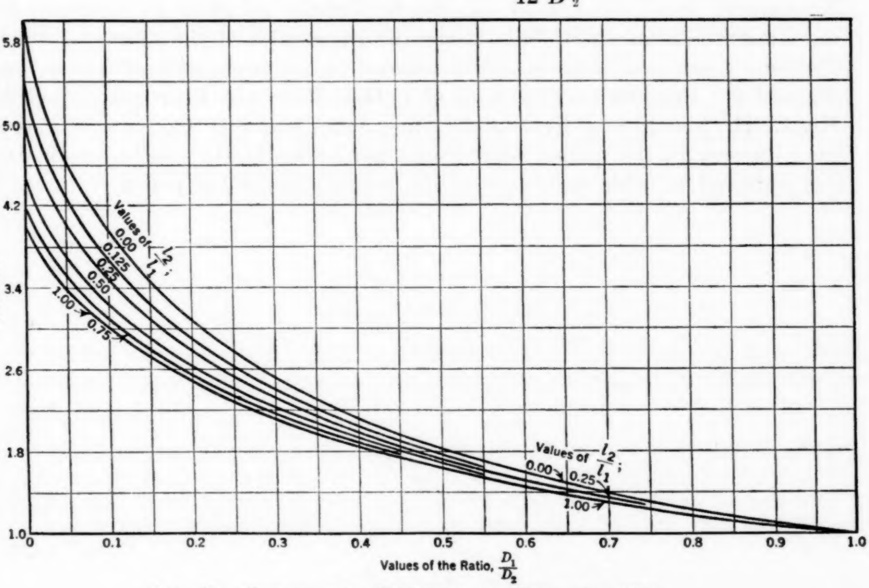

FIG. 19.—CURVES FOR SOLUTION OF EQUATION (39).

The values of X corresponding to different ratios of $\frac{D_1}{D_2}$ and $\frac{l_2}{l_1}$, can be picked from the curves of Fig. 19. It will be noted that X has a minimum value of 1 and a maximum value of 6.

Assuming that K is expressed in meters per day at 10° cent., and at 45% voids, the general seepage formula for any hydraulic-fill dam—at any percentage of voids (within working ranges), for any temperature, in gallons per day—may be expressed approximately by,

$$Q = 24.54\ X\ K\ H^2_2\ (l_1 + 2\ l_2) \left(\frac{D_1 + D_2}{12\ D^2_2}\right)\left(\frac{V}{45}\right)^6\left(\frac{t + 10}{60}\right) \ldots\ldots (42)$$

and substituting $150\ d^2$ for K in Equation (42),

$$Q = 300\ X\ d^2\ H^2_2\ (l_1 + 2\ l_2) \left(\frac{D_1 + D_2}{D^2_2}\right)\left(\frac{V}{45}\right)^6\left(\frac{t + 10}{60}\right) \ldots\ldots (43)$$

in which, Q is the quantity of flow, in gallons per day; K, the coefficient of permeability, in meters per day, at 10° cent. and at 45% voids; V, the percentage of voids varying from 30% to 60%; and the other factors are as previously defined.

Acknowledgment

Credit is given throughout the paper for the data used from other sources. The writer is indebted to: The late Allen Hazen, M. Am. Soc. C. E., who, before his death, read the manuscript (which then consisted of about the first half of this paper), and made useful suggestions; E. E. Lochridge, M. Am. Soc. C. E., Chief Engineer of the Board of Water Commissioners, Springfield, Mass., who, realizing the importance of such a laboratory for the construction of the Cobble Mountain Dam, allowed the necessary funds for the purpose and kept up useful interest in the work with criticisms and suggestions; the Engineering Staff at Cobble Mountain Reservoir, especially Mr. R. H. Alberti and Nathaniel Clapp, Jun. Am. Soc. C. E., who took a keen interest in the subject-matter and not only did the routine work, but also rendered valuable assistance in the preparation of the paper.

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PAPERS

GEORGE WASHINGTON BRIDGE: ORGANIZATION, CONSTRUCTION PROCEDURE, AND CONTRACT PROVISIONS

By Edward W. Stearns,[1] M. Am. Soc. C. E.

Synopsis

For the design and construction of the George Washington Bridge, simultaneously with other large bridge projects, The Port of New York Authority built up a large engineering organization. In its development, and also in arranging the program of the construction work, the basic aim was to secure flexibility, which would permit efficient handling of the complex problems involved and would insure rapid construction. This paper contains a review of the preliminary steps leading to the creation of the engineering organization, and describes briefly the organization and its method of carrying on the work, the construction program developed for the building of the George Washington Bridge, the procedure followed in handling construction contracts, and the outstanding provisions of the construction contracts and specifications.

Preliminary Steps

In the early part of 1925, the Legislatures of the States of New York and New Jersey passed concurrent legislation[2] authorizing The Port of New York Authority,

"To construct, operate, maintain and own a bridge, with the necessary approaches thereto, across the Hudson River from points between 170th Street and 185th Street, Borough of Manhattan, New York City, and points approximately opposite thereto in the Borough of Fort Lee, Bergen County, New Jersey."

By virtue of this legislation the State of New York appropriated $100 000 and the State of New Jersey $150 000 for the preliminary work necessary

Note.—Discussion of this paper will be closed in January, 1933, *Proceedings*.

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[2] Chapter 41, Laws of New Jersey, 1925; Chapter 211, Laws of New York, 1925.

for making borings, surveys, engineering studies, investigations, and for hearings, and all expenses incidental to these activities. This legislation provides that the sums appropriated shall be repaid to the States when the cost of construction has been fully paid for and the debt or debts created for such purpose has been amortized. Both Acts provided, however, that no money should be expended by the Port Authority until equivalent amounts had been appropriated by both States. Accordingly, later in 1925, the State of New Jersey furnished \$100 000 out of its total appropriation of \$150 000, equivalent to that appropriated by the State of New York. An amount of \$200 000 was thus made available to the Port Authority for its preliminary studies.

The Port Authority began its studies immediately and, on March 11, 1926, practically one year after the legislative authorization for the project, transmitted a report[3] to the Governors of the two States. The work accomplished to that date included: (*a*) Comprehensive studies to determine the probable volume of traffic over the bridge and the revenues to be derived; (*b*) topographical surveys, river borings; (*c*) engineering design studies to determine the suitable site, size, and type of crossing and its cost; and finally, (*d*) architectural studies to determine the feasibility of "rendering the bridge a befitting object in a charming landscape." The report set forth eleven conclusions, which may be summarized as follows:

(1) The traffic studies revealed an urgent demand for a crossing for vehicular traffic in the vicinity defined by the legislation, and indicated that the traffic would be of sufficient magnitude to make the undertaking financially feasible.

(2) The general location was well chosen both in regard to topography and feasibility of convenient connections to important local and arterial highway routes.

(3) From the engineering point of view, the construction of the bridge with a single river span of at least 3 500 ft. and a clear height above water of about 200 ft. was feasible in every respect and would involve no extraordinary difficulties, nor hazardous or untried operations.

(4) The suspension bridge would be the most economical type and æsthetically superior to any other type.

(5) Should funds for the construction of the bridge be available in 1927, it was expected that, not later than 1933, the bridge would be open for 4-lane vehicular and bus passenger traffic and for pedestrians.

(6) On the basis of the information available prior to completion of the preliminary studies it was estimated that the bridge could be constructed in an initial stage, and opened to highway traffic at a cost of \$50 000 000.

(7) Depending upon the traffic capacity finally to be decided upon, the bridge could be enlarged later at an additional cost of between \$15 000 000 and \$25 000 000.

(8) Conservative traffic analysis indicated that the bridge would be self-sustaining in every respect from the first year, without unreasonable toll charges on traffic.

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[3] Tentative Report of Bridge Engineer on Hudson River Bridge at New York between Fort Washington and Fort Lee.

(9) Growth of vehicular traffic might justify enlarging the bridge, within ten years after its opening, to 8-lane capacity, and within twenty years thereafter, the entire bond issues for construction cost might be amortized.

(10) Architectural studies indicated that the bridge could be designed to blend harmoniously with the grandeur of its natural setting.

(11) On account of the favorable aspect of the bridge and its urgent necessity, it was recommended that preliminary work be carried to completion, and that the States be asked to appropriate an additional sum of $100 000 for that purpose.

Subsequent to the submission of this report, the State of New York made an extra $50 000 appropriation; the additional $50 000 was provided out of the authorized total appropriation of $150 000 in the original legislation by the State of New Jersey. This brought the funds for preliminary studies to the total of $300 000. With these funds it was possible to conclude the preliminary studies before it became necessary to approach the bankers for financing the construction work. In order to provide for financing the bridge construction, the two States enacted additional legislation[4] which was approved by the Governors on March 10, 1926, and May 4, 1926, respectively. These two Acts, subject to certain limitations, pledged the sum of $5 000 000 from each State, payable in five annual installments of $1 000 000. The money, with interest as and when earned, is to be repaid to the States out of revenues derived from tolls, after operating expenses and debt charges incurred for the construction of the bridge have been met. The legislation further provided that the remainder of the money needed for construction and incidental purposes was to be raised by the Port Authority on its own obligations, secured by the pledge of the revenues and tolls arising out of the use of the bridge. The obligation for moneys so raised constitutes a prior lien on the revenues and tolls.

On December 9, 1926, the Port Authority concluded negotiations with bankers for the sale of $20 000 000 of bonds for construction purposes, out of a total authorized issue of $60 000 000. A further installment of $30 000 000 was sold in the spring of 1930 to complete the financing. The moneys advanced by the two States, with the $50 000 000 derived from the sale of Port Authority bonds, made the total of $60 000 000. The greater part of this sum has been used to bring the project to its present (1932) stage of completion.

Engineering Organization

In 1924, the two States had directed the Port Authority to make preliminary studies for, and to undertake the construction of, two bridges over the Arthur Kill—one between Elizabeth, N. J., and Howland Hook, Staten Island, New York, and the other between Perth Amboy, N. J., and Tottenville, Staten Island. The Port Authority retained Waddell and Hardesty, Consulting Engineers, to make the preliminary design studies for these bridges, under the direction of Mr. W. W. Drinker, then Chief Engineer of the Port Authority. In 1925, it beame evident that, for making the pre-

[4] Chapter 6, Laws of New Jersey, 1926, and Chapter 761, Laws of New York, 1926.

liminary studies for the George Washington Bridge, for directing and supervising the construction of the two bridges over the Arthur Kill, and for the handling of the possible fourth project of a bridge over the Kill van Kull between Bayonne, N. J., and Staten Island, a bridge engineering organization of its own would be advantageous to the Port Authority. Accordingly, O. H. Ammann, M. Am. Soc. C. E., later Chief Engineer of the Port Authority, was employed as Bridge Engineer, and he began immediately the building of an engineering staff. A plan of organization was developed, the soundness of which is attested by the fact that, considerably amplified, it stands to-day, after having successfully accomplished the purposes for which it was developed in 1927. It consisted essentially in separating the engineering work of the Bridge Department into divisions, placing at the head of each division an engineer well qualified by training and experience to handle his particular work. Responsibility was placed upon each division engineer for producing work from that division and co-operating with the heads of the other divisions under the general direction of the Bridge Engineer.

The work was divided into the following five divisions: Traffic Studies, Design, Contracts and Specifications, Construction, and Planning of Approaches and Highway Connections. Selection of the personnel for these separate divisions was made chronologically as the need developed.

Traffic Studies.—The Port Authority has no power of taxation, has no authority to assess for benefit, and, at the time of financing the bridge, it owned no physical properties which in themselves could be used as collateral for loans. All the moneys which it borrowed had to be protected by the revenues to be derived from the tolls. It was essential, therefore, for the Port Authority to make a most careful survey of traffic conditions and from this survey to draw conclusions that could stand the acid test of the inquiries of prospective investors.

The Division for Traffic Studies, including a force of traffic inspectors and analysts, was in charge of the Traffic Engineer. As a rule the inspectors were employed only temporarily for such periods of time and at such particular locations as was necessary to accumulate the needed data. The analysts assembled these data and from them forcasted the traffic that would make use of the facility over a period of years.

The studies involved a comprehensive analysis of the following factors:

(*a*) The volume of vehicular and pedestrian traffic over each of the seventeen Hudson River ferries between the Battery, New York City, and Tarrytown, N. Y.

(*b*) The volume of traffic that could reasonably be expected to be diverted to the bridge from each of these crossings.

(*c*) The volume of new traffic that the bridge could be expected to attract, which involved an estimate of the probable effect of the opening of the Holland Tunnel.

(*d*) The total volume of probable traffic over the bridge for each year for a period of twenty years subsequent to its opening, including consideration of probable effect upon bridge traffic of the possible construction of other new crossings south of 179th Street.

(e) The estimating of probable revenues for each year of the 20-year period.

In order to determine the probable volume of traffic which would be diverted from existing ferry crossings, it was necessary to ascertain the distribution of the traffic over each of the ferries, by finding the origin and destination of each vehicle for sample periods of time so selected that the peak and average traffic conditions were reflected. The peak condition occurs in July and the average condition in October. Variations of traffic between week days and Sundays, and from hour to hour, had also to be considered. Inspectors were placed on each of the ferry-boats throughout the day, and they recorded the type of vehicle (that is, whether horse-drawn or motor-propelled, and whether commercial or pleasure vehicle). They also recorded the carrying capacity of the various vehicles, the number of persons actually carried in each vehicle, the State license, the origin and destination of each vehicle, and the frequency with which the particular vehicle used the particular ferry route. These "clockings" by inspectors were made throughout the months of July, August, September, and October, 1925, a force of fifty-six men being employed on the seventeen ferry routes. Detailed information was recorded for a total of 242 000 vehicles.

Clockings were also made of the traffic passing over the streets and highways at advantageous points, not only in the vicinity of the bridge site, but also at points considerably distant from the bridge, in order to determine the potential capacities of these highways for bridge traffic, because the degree of resultant congestion on these arterial connections would affect materially the flow of traffic to the bridge. These investigations included also a comparative study of the traffic carried by the East River bridges particularly during the peaks of traffic. The records kept by each of the seventeen Hudson River ferries were made available and were investigated as far back as 1914 so that an excellent record of the total river crossing was subject to analysis for a considerable period. These records served to show also the periods of peak traffic and were very valuable in forecasting the probable future traffic, under the improved conditions, which the opening of the Holland Tunnel and the construction of the bridge would create. In connection with these studies, consideration was also given to a comparison of the growth in motor-vehicle registration in the Metropolitan District with the flow of the traffic over the river. This study showed clearly that the inconvenience and insufficiency of the facilities provided by the ferry companies acted as a restriction on the flow of trans-river traffic and led to the conclusion that the provision of more convenient facilities would materially increase this traffic. The accuracy of this observation was conclusively demonstrated after the opening of the Holland Tunnel. Detailed figures of the forecasts derived from these various traffic studies have been presented[5] by O. H. Ammann, M. Am. Soc. C. E.

Because of the importance of the traffic studies as a basis for further vehicular crossings, they have been continued by the Port Authority. As a

[5] "George Washington Bridge: General Conception and Development of Design," by O. H. Ammann, M. Am. Soc. S. E., *Proceedings*, Am. Soc. C. E., August, 1932, p. 986.

result of the information tabulated during the seven years, 1925 to 1932, it is possible to predict not only the future volume of traffic, but also the effect of additional crossings.

A change in the organization for traffic studies was made in 1930, when this division of the work was transferred from the Engineering Department to the Bureau of Commerce of the Port Authority.

Design Division.—Centered in the Design Division was the work of preparing the preliminary design studies, general drawings and layouts, stress calculations and design, detailed contract drawings, estimates of cost, the checking of contractors' shop and working drawings, and work of a similar nature. In order to co-ordinate properly the work which was performed under the various contracts, particular attention had to be given to the preliminary design studies, general drawings and layouts, and to the estimates of cost, so that the work that had not been placed under contract would fit properly with that which had already been placed under contract, and also so that the cost of the entire project would not exceed the estimates upon which the financing was based. The preliminary designs, were frequently modified and the estimates of cost revised accordingly. The contract drawings were usually elaborate in their details and although they were frequently revised as the work progressed, nevertheless, they clearly covered the general character of the work to be performed.

The Design Division was subdivided into two general parts, based on the character of the work to be done. Reporting directly to the Engineer of Design, who was in general charge of the Division, were the Assistant Engineer of Design and the Chief Draftsman. All the general design studies, stress calculations, cost estimates, etc., were placed under the direction of the Assistant Engineer of Design. The preparation of contract drawings, detail studies, layouts, and work incidental thereto was placed under the direction of the Chief Draftsman. The sub-divisions were quite flexible, and men were continually transferred from one sub-division to the other, as the need for the Staff Personnel varied, and not infrequently, when it was convenient to the handling of particular problems, men were working for both sub-divisions at the same time.

Directly subordinate to the Assistant Engineer of Design and the Chief Draftsman were a number of Assistant Engineers, each of whom was especially qualified in some particular phase of work—structural steel design, reinforced concrete design, foundations, highway construction, etc. These men functioned somewhat as "squad bosses" do in usual drafting-room practice; they also superintended the work of groups of individuals. Depending on the work in hand, the individual groups, at any one time, might be engaged on work of similar nature on all the bridge projects, or only on one project, and because of the shifting of men from group to group as occasion demanded, the groups varied materially in number from time to time. Although the effort was made to keep each man engaged in the particular work for which he was best qualified, this specialization was not considered all-important, and through the transference from one group to another, each man was given as wide a variety of work as possible. This policy served to keep up a high

morale among the men; it also gave each man a wider and more intimate knowledge of each structure as a whole, and thus led to more accurate and intelligent solution of the problems in hand. It is recognized, of course, that this form of organization necessitated a higher proportion of supervision and direction but, against this, was balanced a better co-ordination of the various parts of the work with a resultant higher degree of efficiency.

Central Division.—The Central Division, in charge of the Assistant Chief Engineer, who reported to the Chief Engineer, was organized to handle the preparation of specifications and contracts, reports, and general administrative matters of the Engineering Department. Negotiations leading up to, and the preparation of agreements with, the various outside bodies whose interests were allied with, or affected by, the construction of the bridge, were handled by this Division.

Flexibility of organization was also essential in the Central Division, because it was necessary to co-ordinate to a considerable degree the functions of the various Divisions in the Engineering Department and to contact with the other Departments of the Port Authority Staff, such as the Legal Department, Real Estate Department, Treasurer, and Auditor. Routine administrative matters were in charge of an Assistant Engineer who reported to the Assistant Chief Engineer. The preparation of contracts and specifications was placed under the direction of another Assistant Engineer, chosen for his particular experience along these lines. During the preparation of the contracts, close contact was maintained not only with the Division heads, but with the Assistant Engineers engaged in the work in the Design and Construction Divisions, and with those engaged in the inspection of materials. Specifications were invariably prepared in draft form and in sufficient number to submit to all those interested, who were thus given an opportunity to review not only the parts of particular interest to them, but also the entire specification. This policy brought about a more complete understanding of each other's problems as between the men in the office and those engaged on construction in the field; it resulted in broader, more complete and more comprehensive specifications, and minimized discrepancies and omissions between the drawings and specifications.

Progress pictures of the work received special consideration. A photographer was employed on the staff of the Central Division, and full equipment for him was provided as soon as construction work began. He reported, at more or less regular intervals, to the Resident Engineers, and received instructions from them as to what particular operations should be photographed. This policy resulted in producing a complete pictorial record, which the writer feels was thoroughly justified by the magnitude and importance of the work.

Closely allied to the photographic progress record, although handled as an entirely separate entity, is a film history of the construction of the main span of the bridge. These films show the actual construction in the field of the principal operations and have proved valuable in acquainting the public with the progress of the work and the methods used. They constitute a record that should prove of great interest and value to succeeding generations.

Construction Division.—All the preliminary surveys, including topography, triangulations, river soundings, mapping, and supervision of subsurface borings, and, later, the detailed surveys necessary for the preparation of the contract drawings, were made by the Construction Division. This Division also had charge of the supervision and inspection of the field work, including the establishment of the lines and grades upon which the various parts of the structure were constructed, but excluding the inspection of materials. The measurement of the work performed and the preparation of the approximate monthly estimates and the final estimates, upon which the contractor's total compensation was based, also originated in this Division.

After careful analysis of the problems involved in the construction of the George Washington Bridge, the Construction Division, in charge of the Engineer of Construction, was subdivided into three complete and independent sections—each headed by a Resident Engineer who reported to the Engineer of Construction. The division into sections was effected geographically: The New York Section had charge of all work on the New York side of the river; the New Jersey Section had charge similarly of work on the New Jersey side; and the Central Section had charge of the work of finishing the two top panels of each tower and all work on the suspended structure, floor system, roadway, and appurtenances in connection with the bridge proper. The anchorages themselves were under the jurisdiction of the Resident Engineers of the New York and New Jersey Sections, respectively, but all the work on the cables, the floor system, and the roadway, from the easterly end of the New York anchorage to the westerly end of the New Jersey anchorage, was assigned to the Central Section. These various Sections maintained their entities until the bridge was nearly completed, when the Central and New Jersey Sections were merged under the Resident Engineer of the Central Section, in order to release the Resident Engineer of the New Jersey Section for work on surveys of the proposed Midtown Hudson Tunnel.

Only in the case of the erection of the steel towers (both of which were part of a single contract), was there any overlapping of functions between the Sections. The Resident Engineers of the New York and New Jersey Sections supervised the erection of the towers on their sides of the river, the Central Division not being established until after the completion of the first ten panels. No difficulty was involved in this overlapping, however, since the contractor elected to erect both towers simultaneously, with entirely separate equipment and gangs of workmen. As a result, a friendly rivalry developed, not only among the contractor's forces, but also among the Port Authority engineers and inspectors, which was a distinct advantage in producing good workmanship and rapid progress.

The personnel of the Construction Division varied considerably in number, depending on the work under way, but during the greater part of the period of construction, from fifty to seventy-five engineers and inspectors were employed at the bridge site.

Approach Studies.—A special division of the Engineering Department was organized at the beginning of 1929 to handle the general planning of the approaches and the highway connections to both the George Washington Bridge and the Bayonne Bridge. This Division consisted of the Terminal Engineer, who reported to the Chief Engineer, an assistant to the Terminal Engineer, and a number of draftsmen. Comprehensive studies of the traffic needs at the terminals of these bridges were made in close co-operation with the engineering representatives of the municipalities and State organizations that were affected. These studies led to the adoption and approval of approach plans satisfactory to all the agencies concerned, and these plans formed the basis for the agreements with the municipal and State bodies.

In addition to the five major sub-divisions of the Engineering Department, two sub-divisions were created to handle particular work, and, later, these were placed under the general supervision of the other principal divisions. The organization of these sub-divisions as separate units, and their subsequent merging with the other divisions is indicative of the general flexibility in the plan of organization.

Inspection of Materials.—Inspection of all materials used in the Arthur Kill Bridges was placed under the direction of an Engineer of Inspection who built up an efficient organization of masonry material inspectors and metallic material inspectors. A small laboratory, commensurate with the relatively small volume of work involved, was established in rented quarters in Jersey City, N. J., and this organization and equipment proved satisfactory as long as only the Arthur Kill Bridges were under construction. With the start of the work on the George Washington Bridge, however, it soon developed that not only were the facilities insufficient, but that better results could be achieved by separating the inspection of metallic materials entirely from the inspection of masonry materials. Accordingly, two sub-divisions were formed; one devoted to inspection of concrete materials, brick, masonry, etc., under the Engineer of Masonry Inspection; and the other devoted to the inspection of metallic materials of all kinds, under the direction of the Engineer of Steel Inspection. At first, these two came under the direction of the Chief Engineer. In 1929, this arrangement was modified so that the Engineer of Masonry Inspection was placed under the jurisdiction of the Assistant Chief Engineer, and the Engineer of Steel Inspection under the Engineer of Design. This modification was for purposes of organization only, and in no way affected the functional duties of either sub-division.

Each of the Engineers of Inspection had inspectors reporting to him. Those inspecting materials in the New York District were stationed at the laboratory in Jersey City. Others were stationed at the various plants where materials were being manufactured or fabricated. In this way a thorough and complete check of the quality of all materials entering into the bridges was maintained. Only the best quality of material and the highest grade of workmanship were permitted throughout the entire work, and due credit should be given to the contractors who contributed whole-heartedly toward maintaining this standard.

It seems proper, at this point, to describe the modern testing laboratory constructed in Jersey City in 1929, when the less completely equipped and smaller laboratory became too congested for further efficient work. Although, of course, it is not a part of the George Washington Bridge organization, as such, its functions are closely allied to those of the sub-divisions for the inspection of materials.

The Port Authority Laboratory building is 50 by 100 ft. in plan. It consists of two stories and basement, with a higher portion on one end to provide housing for a 1 000 000-lb. testing machine. The building is of structural steel, on concrete foundations, and has brick walls backed with tile, concrete floors, and tile partitions.

It contains facilities for routine specification acceptance tests for concrete and for all the check tests on structural steel, and it also has facilities for conducting some research work. In the main testing room, in addition to the 1 000 000-lb. machine, is a 50 000-lb. testing machine that is used for smaller concrete test specimens and for machine tests for structural steel and castings. A view of this equipment is shown in Fig. 1. This main testing room is arranged so that trucks can be driven into it from the street, in order that heavy pieces may be lifted from them by an overhead traveling crane. In addition to the main testing room, there is a machine shop, a chemical laboratory, and concrete, cement, and aggregate laboratories. Fig. 2 shows a general view of the chemical laboratory. In the basement there is a moist room for curing concrete specimens, with equipment for maintaining constant temperature and humidity. The building also contains office space, drafting-room, and inspectors' rooms.

Research and Tests.—Because of the unusual magnitude of the undertakings on which the Port Authority has been engaged, it was deemed advisable to establish a Division of Research, the purpose of which was to make scientific investigation of important problems on which it was felt that there might be a lack of published information. It is not the purpose of this paper to treat of the work done in this Division, since it will be described in other papers.

In brief, the functions of this Division were the making of stress measurements in the towers and in the anchorage steel of the George Washington Bridge, the investigation of full-sized specimens representative of certain tower sections, and of certain sections of the Bayonne arch, and the making of stress measurements on the Bayonne arch. Some of the work of this Division was done in co-operation with the National Bureau of Standards, in Washington, D. C. This Division was headed by an Engineer of Research and Tests, who at first reported directly to the Chief Engineer, and, later, to the Engineer of Design. Under the Engineer of Research and Tests was a group of assistants chosen particularly for their interest in research matters. These men not only carried on the actual work, but developed lines of investigation particularly suited to the purposes for which the Division was organized. They designed, and in many cases made, special instruments for obtaining the results they were seeking.

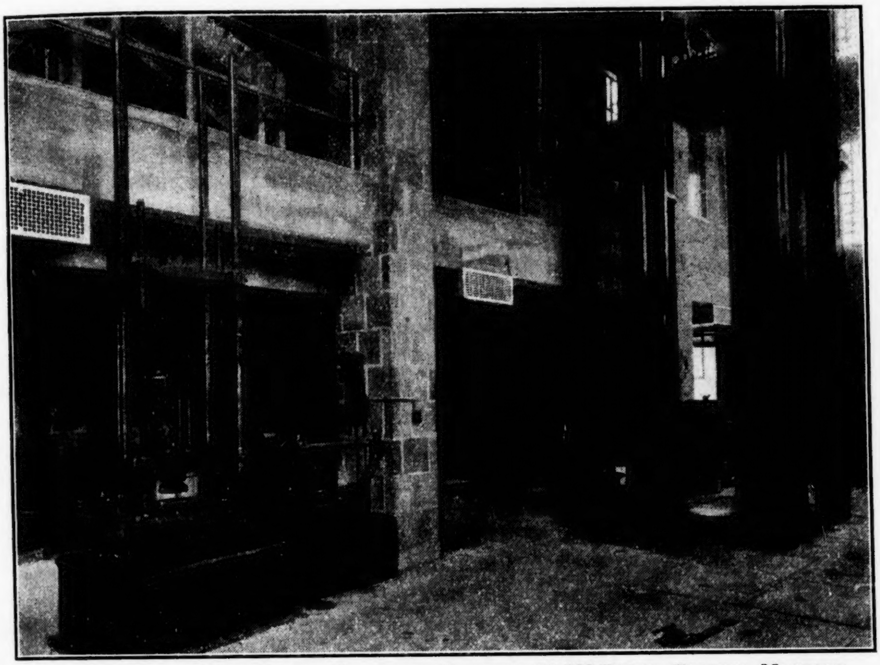

FIG. 1.—VIEW OF LABORATORY BUILDING, SHOWING 50 000-POUND TESTING MACHINE.

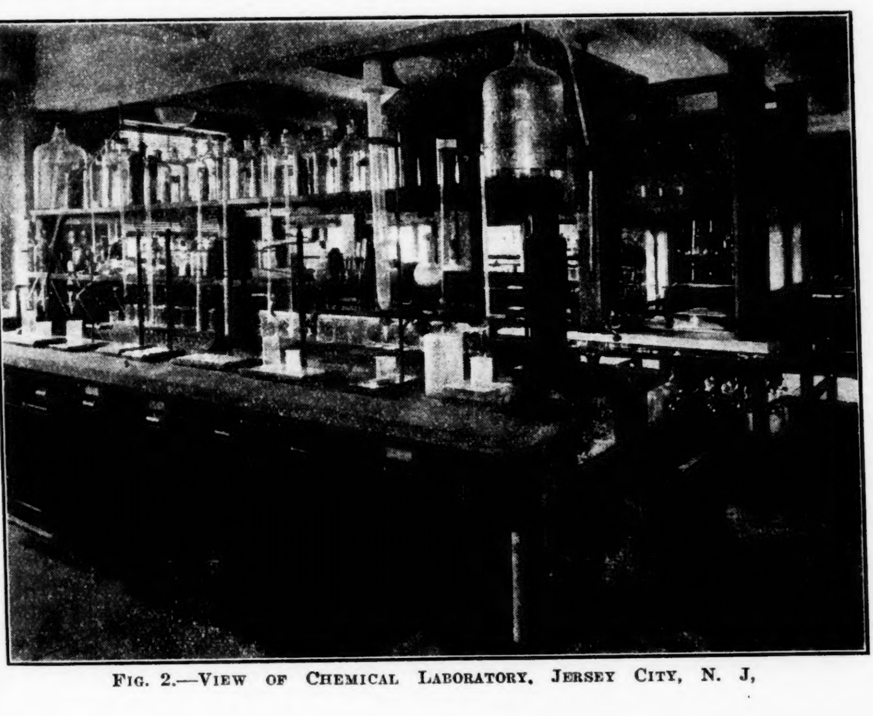

FIG. 2.—VIEW OF CHEMICAL LABORATORY, JERSEY CITY, N. J,

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Program of Construction

After comprehensive study of possible methods, it was decided to divide the construction work into a number of separate contracts, the plans and specifications for each such part being prepared as the progress of the work as a whole required. In so far as separation between the main span and approaches was involved, division was quite essential in any event. The State laws authorizing the construction of the bridge provided that "the plan of the approaches at either end of the bridge shall be subject to the approval of the respective Governors of the States of New York and New Jersey and of the respective municipalities in which they shall be located." Such complete and comprehensive studies were necessary for the solution of the approach problems that the construction of the main span would have been delayed several years had it been necessary to defer it until the approach plans had been definitely settled. It was deemed advisable, however, not only to make a separation of the work as between the approaches and the main bridge, but to let separate contracts for the various integral parts of the structures as the work progressed.

The advantages of this method of procedure are briefly, as follows:

First.—In such a large project it is essential that an extended study of the various parts be continued after the project has been financed. Thus, study of certain parts can be deferred with advantage and, if conditions arise that make it necessary or desirable, certain parts can be submitted to restudy and possibly changed more or less radically.

Second.—By preparing detailed contract drawings and specifications as the work progresses, more thorough and elaborate treatment can be given to these papers, and, at the same time, much more rapid progress can be made in construction because certain parts of the work can be started prior to the elaboration and completion of studies on the entire project. An actual example of these advantages is given by the case of the New Jersey anchorage and the contract for the bridge cables. At the time bids were received for the New Jersey anchorage final decision had not been made either as to whether the cables should be of eye-bar chains or steel wires, or if wire cables were used, each pair of cables would have one cable placed above the other, or would have the cables side by side. Upon decision of these questions depended the ultimate dimensions of the anchorage tunnels and pits. However, it was anticipated that, by the time the contractor for the anchorage was ready to start excavation of the anchorage tunnels, decision on these questions about the cables would have been reached. It was possible, therefore, to show, on the contract drawings, the average dimensions for the tunnels, upon which the contractor could base his unit price for the excavation. This permitted him to finish much of his work before the final determination of dimensions had been reached. The anchorage work was thus begun at a sufficiently early date to expedite construction of the bridge as a whole.

Third.—This procedure eliminates the difficult and uncertain task of developing a definite time schedule for all parts of the work at the outset and, at the same time, permits taking advantage of any saving of time effected on

particular parts of the work. It avoids any conflicts and litigation that might arise because of delays.

Fourth.—By avoiding the need of awarding contracts for construction work, the performance of which could not be begun for several years thereafter, considerable economy is possible through relieving bidders from the necessity of protecting themselves against uncertain changes in prices of materials or labor that may develop during the intervening period.

Fifth.—The award of contracts for the separate parts of the work left more time available for the acquisition of the necessary real estate. This additional time permitted negotiations for more favorable purchase prices, and, in the case of improved real estate, permitted its continued use subsequent to purchase and up to the time it was necessary for it to be turned over to the contractors, for producing revenues and thus lightening the total financial burden.

As soon as the financing of the project was assured, a careful study was initiated to devise a program of construction that would permit the fullest competition in bidding, would result in the most logical sub-divisions of the various elements of the work, and would occasion no delay in the starting of various parts, or in the progress of the work as a whole. The divisions of the work, and the construction contracts for each division, were arranged in logical classification as to nature and location. For example, the New Jersey tower required deep foundations, entirely different in character from those for the New York tower, whereas the New York tower foundations, both in the nature of the work and in geographical location, were closely allied to the work for the New York anchorage. Accordingly, one contract covered the New Jersey tower foundation work, whereas another combined in a single contract the New York anchorage and the New York tower foundations. Again, the New Jersey approach and the New Jersey anchorage both required the excavation of rock in the Palisades and could be included in a single contract. The steelwork for the bridge proper could be handled either as a single contract or as two or more contracts. The work on the approaches could logically be subdivided into contracts classified as to the nature and location of work.

At first, the studies of classification were made only for the earlier and major elements of the work, the schedule of the later operations being deferred until the details of design could be given further study and until more accurate determination could be made of the work involved. This deferring of problems that did not require immediate attention made more time available for careful analysis and solution of those involved in the earlier work. After determining the classification into contracts, attention was given to an analysis of the major operations involved in each contract, in order to allow the proper period of time for their completion. Finally, consideration of the problem as a whole led to determination of the approximate date at which each contract would have to be started in order not only to have it begin immediately upon completion of the next preceding contract, but also to have it completed by such date as to provide ample time for the next succeeding

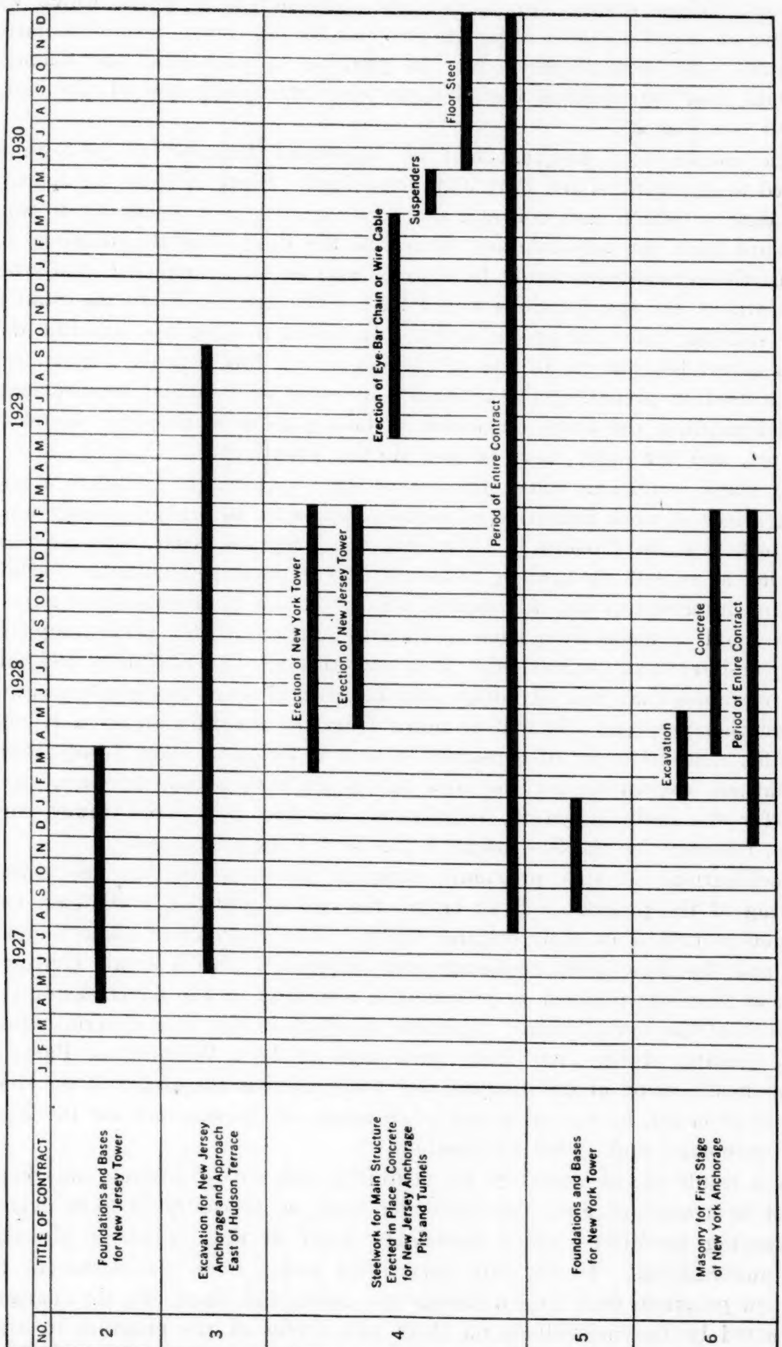


FIG. 3.—FIRST CONSTRUCTION PROGRAM.

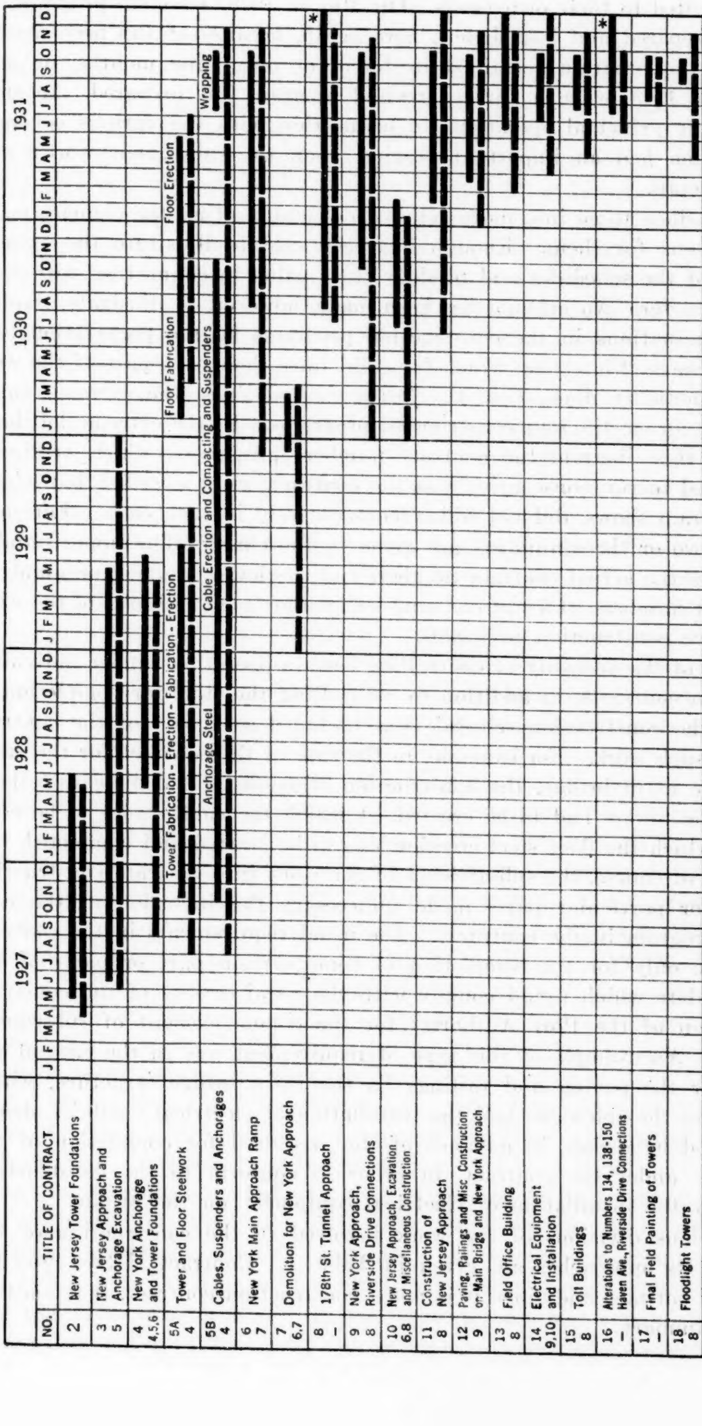
contract. These studies led to the first construction program, which was adopted on April 1, 1927, although previous to that time, several tentative schedules had been prepared for the purpose of arranging the financial program, and determining the probable date for completion of the entire project (see Fig. 3).

The construction program had an important function to perform as applied to the work of the Port Authority Staff. Since it fixed the approximate date on which each contract should be started, it was possible to work backward from this schedule and determine the time when the drawings for any particular contract would have to be started and completed, and when the drafts of the specifications would have to be distributed to the Staff to allow sufficient time for proper study. In short, it became a schedule that had a direct bearing on all the operations in the Engineering Department. It was used in preparing the necessary forecasts of financial requirements, for determining the dates on which certain parcels of property would be required, and for other more or less similar functions.

So many conditions affect the actual time required to complete a particular piece of work that it is, of course, impossible to predict exactly when the work will be finished. Therefore, the program itself, was properly regarded as something flexible, although it was developed with the various elements intermeshed into a complete whole. It was planned so that adjustments could be made from time to time to compensate for work completed sooner or later than anticipated. It is only through this flexibility and constant vigilance that full advantage can be realized from any predetermined construction program. It will be noted from the program of April 1, 1927 (Fig. 3), that the order of construction was as follows: New Jersey tower foundations, excavation for the New Jersey anchorage and approach, steelwork for the main structure, foundations for the New York tower, and, finally, masonry for the first stage of the New York anchorage.

Modifications of this program illustrate the flexibility in the actual handling of the program. Even before the end of 1927, only the first two contracts remained in their original order. The New York tower foundations and the New York anchorage were combined into a single contract, which at first was planned to precede the awarding of the contract for the steelwork of the main bridge. Because of delay in the final determination of the placing of the New York anchorage in Fort Washington Park, a further modification of the program was made, so that the award of the steel contract preceded, by several months, the award of the contract for the New York anchorage and tower foundations.

As a result of the necessity for flexibility, the entire program was kept at first in a state of flux, changes being made as necessary to meet delays and varying conditions which invariably occur at the beginning of such large undertakings. It was felt inadvisable under these circumstances to issue new programs each time a change was made, and, therefore, the changes were noted by the individuals on their own copies of the program in any



Notes: * Construction on Contracts Numbers 8 and 16 Continued into 1932.
Contracts for Completion of 178th St Approach Tunnel not Shown.
Solid Lines Indicate Actual Period of Construction Work. Broken Lines show Maximum Limits of Program for Various Contracts as shown on all Programs Throughout the Work.

FIG. 4.—COMPOSITE CHART OF CONSTRUCTION PROGRAMS.

way best suited to their own needs. On May 5, 1928, a second program was formally prepared and issued, but, here again, because of the necessity for flexibility, the program remained in force for only nine months, or until February 5, 1929, when a further revised program was prepared. In these programs the principal operations in connection with the various contracts (all of which had an important bearing upon the subsequent work) were shown in detail.

It is needless to go into more detailed discussion of all the changes made, or the reasons for them. Enough has been said to illustrate the extreme flexibility of the schedules and to show their value in connection with work of this character. An attempt has been made, however, to illustrate graphically the fluctuations in the construction programs by the preparation of a composite chart (Fig. 4) on which the solid lines show each part of the work as it was actually done, and, the heavy dashed lines, the extreme variations as set up on different construction programs. In the extreme left-hand column of this diagram the contract numbers are given, which originally were planned to run consecutively as the contracts were awarded, but which, as the diagram shows, did not follow consecutively in each case. For some contracts, two or three numbers are given in the chart. The upper number is invariably the actual contract number, and the lower number, or numbers, are contract numbers which at one time or another were assigned to the work in successive construction programs.

Because of the necessity of controlling certain operations under individual construction contracts, in addition to controlling the time for completion of the work, the construction schedule was included as a part of the contract papers for such work. For example, in the case of the contract for the steel-work of the main bridge, the construction schedule provided definite dates at which the towers had to be erected, at which the cables were to be completed, at which the floor steel erection was to be started and completed, etc. In general, of course, the sub-division of the work into separate construction contracts for particular parts made unnecesary the inclusion of the construction program in the contracts. The usual requirements in the contract papers were only for the completion of some certain part or parts of the work by a date which would comply with the requirements of the construction program of the Port Authority for the commencement of subsequent operations. An example of this type of requirement was in the case of the contract for the paving and railings on the main bridge structure, which required that the sidewalks and the installation of electrical conduits should be completed by a date in advance of that required for completion of the entire work under the contract, in order to conform to the construction program for the installation of electrical equipment on the bridge.

The various divisions of the work involved in the construction of the bridge and its approaches are given in Table 1. Contract HRB-1 was for preliminary borings, and since it is not a construction contract, it is omitted from this summary.

Construction Procedure

All the construction contracts were let on the basis of competitive bids, the award being made in each case to the contractor who, in the judgment of the Port Authority, qualified as the lowest responsible bidder. Responsibility was determined by financial ability, experience, equipment, and organization to do the work. In most cases, the lowest bidder proved to be satisfactory, but occasionally it was deemed to be to the best interests of the Port Authority and the public to let the contract to one of the higher bidders.

The contracts were in two different types, which came to be known, respectively, as the "long" form and the "short" form. The short-form contract was developed from the long form, for use for comparatively simple work involving relatively small expenditures. Neither type, however, was set up with specific provisions or phraseology as "standard" to be used in every case, but each contract was examined critically, both by the Engineering and the Legal Departments of the Port Authority, in the light of the particular work to be performed, and of past experience and general policy, and modifications both in the substance and the wording were made from contract to contract.

The intent of the short-form contract was to minimize the routine in awarding the contract. The long-form contract provided a proposal that was an offer to enter into the contract for the work; its acceptance created an agreement for the execution of the contract but not for the performance of the work. The short-form contract on the other hand was a letter form of proposal for performing the work and its acceptance effectuated the contract. Contracts in the short form were not publicly advertised; no cash deposit was required for the issuance of the contract papers; and no certified check was required with the proposal. The contract papers were sent to prospective bidders who were invited to submit proposals in duplicate on or before a definite date, generally without public opening. The successful bidder was required to furnish a surety bond before the return to him of one copy of the proposal with the Port Authority's acceptance endorsed thereon.

Except under unusual circumstances, contracts in the long form were advertised in engineering periodicals and in daily papers selected to give wide distribution throughout the Port District. In addition, letters calling attention to, and enclosing a copy of, the advertisement were sent to contractors likely to be interested in the particular work, and to agencies whose business it is to keep contractors advised of prospective work. The advertisement always described the work briefly, gave information for obtaining the contract papers and the deposit required for them, and stated the exact time and place for public opening of bids.

The proposals were opened promptly at the appointed hour, and no bids were received that were delivered after the opening had begun. As promptly as practicable after the opening, a conference was arranged for the lowest bidder with the Engineering Staff, and, except in cases where the lowest bidder was of known responsibility, conferences were also arranged with the second, third, and even the fourth low bidders. At each of these conferences,

TABLE 1.—SUMMARY OF CONTRACTS FOR THE GEORGE WASHINGTON BRIDGE

Contract No. (1)	Description of work (2)	Dates of advertising work, opening bids, and awarding contracts, respectively (3)	Work completed (4)	Approximate cost (5)
HRB-2....	New Jersey Tower Foundation — two granite-faced piers, containing 36 000 cu. yd. of concrete and 1 400 cu. yd. of granite	Mar. 7, 1927 April 11, 1927 April 15, 1927	May 29, 1928	$1 059 000
HRB-3....	Excavation for New Jersey Anchorage and Approach — approximately 220 000 cu. yd. of trap-rock	May 12, 1927 May 31, 1927 June 2, 1927	May 25, 1929	1 150 000
HRB-4....	New York Anchorage and Tower Foundations — 9 700 cu. yd. of concrete in two tower bases; 107 000 cu. yd. of concrete for first part of anchorage	Feb. 9, 1928 Mar. 5, 1928 Mar. 8, 1928	Mar. 13, 1929	1 088 000
HRB-5*A*...	Towers and Floor Steel — Fabrication and Erection of Structural Steel — 40 000 tons for two towers and 17 000 tons for main bridge floor system	Aug. 4, 1927 Oct. 3, 1927 Oct. 13, 1927	Jan. 26, 1931	10 760 000
HRB-5*B*...	Cables, Suspenders and Anchorage Steelwork — manufacture and erection of four wire cables weighing 30 000 tons and 1 300 tons of 2¼-in. suspenders, fabrication of 5 000 tons of structural steel for the anchorages and erection of the steel in the New Jersey Anchorage	Aug. 4, 1927 Oct. 3, 1927 Oct. 13, 1927	Oct. 23, 1931	12 193 000
HRB-6....	Main Approach Ramp, New York — concrete arch over Riverside Drive, concrete and steel viaduct and miscellaneous construction in adjacent streets	June 19, 1930 July 14, 1930 July 17, 1930	Oct. 23, 1931	966 000
HRB-7....	Demolition and Removal of Buildings for the New York Approach — between 178th and 179th Streets, west of Fort Washington Avenue	Oct. 31, 1929 Nov. 18, 1929 Nov. 21, 1929	Feb. 27, 1930	150 000
HRB-8....	Vehicular Tunnel in West 178th Street of New York Approach — construction of surface and sub-surface plazas between Pinehurst and Fort Washington Avenues and tunnel structure (except finish, pavement, and lighting) from Fort Washington Avenue to Amsterdam Avenue	June 19, 1930 July 14, 1930 July 17, 1930	Nov. 1, 1931*	2 138 000
HRB-9....	Riverside Drive Connections of New York Approach — construction of roadways and reinforced concrete arch bridge	Aug. 21, 1930 Sept. 15, 1930 Sept. 18, 1930	Oct. 23, 1931	1 250 000
HRB-10...	New Jersey Approach Excavation and Miscellaneous Construction — excavation principally in rock and rough grading, and construction of walls and abutments	May 8, 1930 June 2, 1930 June 5, 1930	Jan. 8, 1931	323 000
HRB-11...	Paving and miscellaneous construction for the New Jersey Approach	Dec. 11, 1930 Jan. 5, 1931 Jan. 8, 1931	Oct. 27, 1931	565 000
HRB-12...	Paving, railings and miscellaneous construction on Main Bridge and New York Anchorage	Feb. 5, 1931 Mar. 2, 1931 Mar. 5, 1931	Oct. 23, 1931	493 000
HRB-13*A*..	Field Office Building in Fort Lee — a steel framed, masonry wall building for housing operation and maintenance forces and equipment	April 23, 1931 May 18, 1931 May 21, 1931	Dec. 30, 1931	195 000
HRB-13*B*..	Heating and ventilating equipment for Field Office Building	April 23, 1931 May 18, 1931 May 21, 1931	Dec. 20, 1931	13 000

* Essentially complete.

TABLE 1.—(*Continued*)

Contract No. (1)	Description of work (2)	Dates of advertising work, opening bids, and awarding contracts, respectively (3)	Work completed (4)	Approximate cost (5)
HRB-13*C*..	Electrical installation in Field Office Building	April 23, 1931 May 18, 1931 May 21, 1931	Jan. 8, 1932	$5 000
HRB-13*D*..	Plumbing system for Field Office Building..	April 23, 1931 May 18, 1931 May 21, 1931	Jan. 11, 1932	6 000
HRB-14...	Electrical Equipment and Installation on Bridge and Approaches — (except Riverside Drive connections)	May 7, 1931 May 25, 1931 May 28, 1931	Feb. 26, 1932	92 000
HRB-15...	Toll buildings..........................	July 19, 1931† July 16, 1931 July 27, 1931	Oct. 23, 1931*	155 000
HRB-16...	Alterations to certain buildings encroaching on Riverside Drive Connections	June 4, 1931 June 22, 1931 June 25, 1931	Mar. 24, 1932	80 000
HRB-17...	Final field painting of towers............	July 23, 1931† July 31, 1931 Aug. 1, 1931	Dec. 11, 1931	27 000
HRB-18...	Flood Light Towers, New Jersey Plaza—four steel towers mounting eight flood lights each	Aug. 20, 1931† Aug. 31, 1931 Sept. 10, 1931	Jan. 16, 1932	41 000
Total approximate cost of construction done under contracts		....	....	$32 752 000

*Essentially complete.

† Not advertised; contractors requested to bid.

the work was thoroughly discussed and every effort was made to be sure that the contractor had acquainted himself thoroughly with the work and was satisfied with the sufficiency of his bid, as well as to determine his fitness to perform the work. Each prospective contractor, unless of well-known reputation, was required to furnish a number of references as to performance (which were later investigated by the Engineering Staff), and to furnish financial statements and banking and credit references, which were investigated by the Assistant Treasurer of the Port Authority.

Following the investigations, the Chief Engineer reported the findings, together with a recommendation as to award of the contract, to the General Manager of the Port Authority. The report was then transmitted to the Port Authority Commissioners who, by appropriate resolution, authorized the General Manager to execute the contract. As soon as such resolution became effective, the successful bidder was notified by the General Manager of the award and was furnished with duplicate copies of the contract forms, prepared for signature by the contractor. The return of these copies, properly executed, together with security for performance, was required within seven days. Upon their return, they were signed by the General Manager, thus effectuating the contract, and one copy was delivered to the contractor. This routine generally required three to four weeks from the date of the

receipt of bids, and therefore in some cases, where prompt performance was required, the procedure was varied, and the Commission authorized the General Manager, in advance, to make an award and execute the contract.

The procedure during performance of work under the contract was not materially different from the usual construction practice except as it was affected by the complexities of a large organization engaged, not in one great undertaking alone, but in several. The Chief Engineer was the responsible head, of course, but he could not keep personally in touch with all details under all contracts. Direct contact with the contractor was, as far as possible, through the Engineer of Construction, the Engineer of Design, and the Assistant Chief Engineer. The Chief Engineer, because of his heavy responsibilities and duties, necessarily placed wide discretionary responsibility upon the various Division heads. The contracts differentiated, by definition, between "Chief Engineer" and "Engineer," and the two words were used throughout with the purpose in view of reducing to a minimum the acts to be required by the Chief Engineer. The definitions provided that "Chief Engineer" meant the Chief Engineer (or in the event of his absence or disability the Assistant Chief Engineer) acting personally, and that "Engineer" meant the Chief Engineer acting either personally or through his duly authorized representatives.

All questions involving additional expenditures (extra work, adjustments of prices, changes in plans, delays, etc.) and all matters affecting the contractual relations were left to the decision of the Chief Engineer. Questions pertaining to performance of the work, such as quality of materials, workmanship, and orderly progress, were left to the authority of the Engineer. The Chief Engineer also (as is customary in contracts for public work) acted in the dual capacity of representative of the Port Authority, in charge of all operations, and as arbitrator in cases of dispute between the Engineer and the contractor.

Upon completion of the work under a contract, a detailed final inspection was made, usually by the Chief Engineer himself accompanied by one or more of the Division heads and by the Field Engineers directly in charge of the work. Usually, upon the completion of the work, there were a very small number of claims by the contractor for extra compensation, which had to be considered and adjusted. Furthermore, the total quantities involved in work under unit-price contracts were, of course, carefully checked, and agreement upon them was reached with the contractor. Following this procedure, the Chief Engineer issued a final certificate showing the entire amount of work performed and the compensation earned by the contractor.

In accordance with the requirements of the contract, the contractor was then required to furnish a detailed sworn statement of all outstanding liens, claims, and demands, just and unjust, of sub-contractors, material men, laborers, and third persons, and satisfactory evidence that the work was fully released from all such liens, claims, and demands. The contract provided that final payment to the contractor should act as a release to the Port Authority of all claims; and, as a matter of completing accounting records, each contractor was required to file a separate form of release in confirmation

of this contract provision, when final payment was made. At the time of preparing the final certificate, the Chief Engineer submitted to the General Manager a report upon completion of the contract, and, upon the transmittal of such report to the Commissioners, an appropriate resolution was adopted authorizing the final payment.

Contract and Specification Provisions

Considerable study was given to the preparation of the basic form of construction contract so that it would be clear and definite in its provisions and so that it would place full responsibility upon the contractor for doing his work and for damages resulting from his operations, but at the same time would protect the contractor as far as practicable against losses from unforeseen conditions. Some of the outstanding and unusual provisions will be discussed briefly in the order in which they occur in the contract papers.

Information for Bidders.—The Information for Bidders required each bidder to submit with his proposal a detailed list of plant and equipment, and a detailed description of the method and program of work he proposed to follow. This was not intended as something to which the contractor would be rigidly held, but rather as an indication to the Port Authority Staff of the fitness of the contractor and the thoroughness with which the problems had been studied. In this requirement, it was stated that the information would be regarded as confidential, but nevertheless it was sometimes found that considerable questioning was necessary before complete information could be elicited from the bidder. In spite of this reluctance of some bidders to disclose their plans, the writer believes that this provision is valuable to the determination of the bidder's responsibility.

Each bidder was required to deposit a certified check with his bid. The amount required for each contract was determined by judgment based on consideration of the work to be done, and not according to a fixed percentage of the estimated cost. A much higher percentage was required on the smaller contracts. The amount of the check ordinarily varied from 2% to 6% of the estimated cost of the work. The aim was to make it sufficiently substantial to afford a valuable indication of the contractor's financial ability; and investigation of the contractor's financial statements served to show whether the check was furnished by him or by a surety company.

On the greater part of the contracts, each bidder was required to submit an agreement with his proposal, signed by the surety that he proposed to have execute the performance bond. This agreement provided both that the surety would execute such a bond in case the contract was awarded to the bidder and also that, in case the bidder failed to execute the contract, the surety would pay to the Port Authority the liquidated damages set up in the Information for Bidders for such event. This "Agreement of Proposed Surety," and the certified check, provided the Port Authority with a double protection in case the low bidder should default and refuse to execute the contract.

Toward the end of the work, the Port Authority decided that this double protection was unnecessary and since, of the two forms, the deposit of the

certified check was considered the more valuable, the agreement of the proposed surety was abandoned. In all contracts, except the short form, the contractor was permitted, in lieu of the surety bond, to deposit acceptable security duly transferred or assigned to the Port Authority.

In unit-price contracts, the Information for Bidders contained a requirement that the successful bidder should furnish "an analysis of bid prices" when he returned the contract forms for signature by the Port Authority. This requirement proved helpful, and it is believed that it acts as a deterrent against the unbalancing of bid prices. The analysis was intended to be a complete "break down" of each unit price, to be used as a guide for the determination of an equitable price when, because of changes in character or quality of the work, it became necessary to make an adjustment. An analysis was not generally required in lump-sum contracts because it would be so complex as to be of little aid in any case of an adjustment of the lump-sum compensation.

Form of Contract.—A differentiation between the contract proper and the specifications was considered desirable, and accordingly the contract provisions proper were grouped in a section entitled "Form of Contract." All provisions to establish contractual relationships were included in this section and they could be changed only by the written consent of both parties. The specifications contained all provisions as to performance of work, materials, workmanship, and such conditions as were subject to change by the Chief Engineer, without modification of contractual relationships. The Form of Contract gave specific authorization to the Chief Engineer to modify the contract drawings and specifications, to require performance of work not shown on them, or to countermand directions for parts of the work.

Except in a few cases, unit-price contracts were used throughout because of their greater flexibility. The Port Authority contract, however, differs from the form commonly used in that it is not separable; that is, the prices paid under the separate items were not payments made for, or because of, work pertaining solely to those particular items, but were prices applied to the quantities of certain classes of work actually performed. When these prices were multiplied by the actual quantities involved in their respective items and the products were added, the total compensation to the contractor for the original contract work was obtained. The contractor was expected to include in the prices of these items—in any manner that he considered best—the cost of all work involved in the execution of the contract as a whole. The result of this provision in the unit-price contract was that it was unnecessary to provide a specific item for each class of work to be performed, and items were provided only for the principal classes of work, such as excavation, concrete, structural steel, and masonry, and for all classes of work, the amount of which was likely to vary from that originally estimated.

Extra work was defined in the Form of Contract as work required by the Engineer in addition to that required by the contract drawings and specifications which, in the judgment of the Chief Engineer, differed in general character from the work pertaining to the items under which unit prices were quoted. The Chief Engineer was authorized to make agreements with the

contractor for unit-price or lump-sum compensation for extra work, or to order the work performed on a basis of cost plus percentage. Compensation for cost plus percentage was the actual net cost in money to the contractor of materials and wages of applied labor (including premiums on Workmen's Compensation Insurance) plus 15%, and plus such rental for plant and equipment as the Chief Engineer deemed reasonable.

In order to insure fairness in compensating the contractor in the event that changes in plan or unforeseen conditions developed wide variation in quantities of work pertaining to any item, or involved changes in the character or quality of such work, the Form of Contract contained a provision giving the Chief Engineer authority to increase or decrease any of the unit prices quoted to such extent as, in his judgment, would effect the proper modification in the compensation warranted by such change in the work. In the case of such modifications, the Chief Engineer was also authorized to agree with the contractor upon lump-sum adjustment in lieu of adjustments of the unit prices. In the case of foundation work in particular, where the quantities may vary considerably from those estimated, this provision makes for fairness and equity because it provides for proper distribution of cost of plant and equipment, through adjustment in the unit prices.

The contracts did not provide any bonus in the event of completion before the specified date, but they did provide heavy liquidated damages for failure to complete within the time limits set up in the contract. In the handling of work of the magnitude of the George Washington Bridge under a program of construction—with numerous separate contracts for the various parts of the work—little, if any, advantage is obtained from bonuses for early completion. It is conceivable that even if full advantage could be taken of the time saved by early completion of some contracts, the aggregate of the bonuses paid therefor might be out of all proportion to the advantages gained.

It is essential to successful operation under a construction program that the various parts of the work be properly synchronized. While it is true that every opportunity must be taken to advance the completion of the entire work, nevertheless, a bonus on early completion of individual contracts might result in speeding up, out of all proportion, such parts of the work as could be easily expedited. This would result in certain parts being completed in advance of certain other parts just as essential, and no advantage would accrue from the bonus payment on the work completed ahead of schedule. For example, the contract for the cables was placed with one contractor and those for the two towers and the two anchorages with three other contractors. It is evident that any one of these three other contracts, under the stimulus of a bonus award, might have been completed far ahead of the others with no resultant advantage to the Port Authority, because the cable work could not be started prior to the completion of all of them.

Conversely, the beginning of the cable work would be delayed by the failure to complete on time any of the other three contracts. This latter condition was guarded against by liquidating the damages resultant therefrom in a very material sum. Such large sums of money were invested, and the prospective revenues from the bridges were so great that it was impossible

to liquidate fully the possible damages that might result by reason of delay, and, therefore, each contract was carefully considered by itself and the liquidated damages fixed in as large an amount as the contract would reasonably permit.

In many of the contracts, special provisions were made with regard to monthly payments to the contractor. It was expressly provided in the Form of Contract that the monthly payments were to be made as advances to the contractor to assist in financing the work. The sum retained out of each such monthly advance was regarded as security to the Port Authority for insuring performance of the work and protection against possible errors in the preparation of the monthly estimates. Inasmuch as the rate of interest on money borrowed by the Port Authority was generally less than the rate that would be paid by a contractor, the effort was made, in the interests of economy in the work as a whole, to keep the retained amount at the smallest value consistent with proper protection. This was accomplished in several ways. For example, in Contracts HRB-5*A* and HRB-5*B* (Table 1) provision was made that 10% of the monthly estimates would be retained at first, but that when the value of the work performed was equal to one-fourth the total contract price, only $7\frac{1}{2}$% would be retained. When the value of the work reached one-half the total contract price, 5% would be retained; and when it reached three-fourths the total contract price, only $2\frac{1}{2}$% would be retained. In other contracts, provision was made that 10% of the monthly estimate would be retained until the value of the work performed was one-half the total contract price, and thereafter the fixed sum of 5% of the total contract price would be retained. In contracts involving large sums of money the smaller percentages afford adequate security, and it is the writer's opinion that such provisions should be the usual practice in contracts for public work.

Unusual conditions affected the making of monthly payments on the contracts for the cables and for the towers and floor steel. The Port Authority could not profit by having a part of the work under these contracts completed far in advance of the date in the construction program (as, for example, one tower); nor by having the fabrication of the cable wire proceeding too rapidly in advance of the tower work; nor that of the floor system too far ahead of the completion of the cables. Since the amount of money invested in these parts of the work was so large and the interest charges on it correspondingly heavy, it was desirable to hold the contractors as closely as possible to the program established in the contract. This was accomplished by providing that the monthly estimates should in no case exceed a sum obtained by dividing the total contract price by the period of duration of the contract and multiplied by the total elapsed time, even if the work was performed with such rapidity that in many cases, the value of the work accomplished was considerably in excess of the monthly estimates. This provision worked no injustice to the contractor, because, inasmuch as the limitation schedule was known in advance, its effect was fully discounted and the greater speed of construction was adopted because the savings in labor and use of plant were of such magnitude that they justified fully the increased financing burden placed upon the contractor.

As has been previously stated, full responsibility was placed upon the contractor for his work and for all damages and claims for damages resulting therefrom. The contractor was the insurer of the Port Authority against all contingencies arising out of the performance of the work; and he was required under the contract to warrant that he had financial ability, that he was experienced and competent, that he was familiar with rules and regulations affecting the work, that he had examined the contract drawings and specifications and the site, that the work could be performed satisfactorily, and that there was no collusion or fraud in connection with the contract. In short, the Port Authority placed full dependence upon the contractor to produce the finished work for the compensation named in the contract, and all costs and expenses involved in the work were his obligations. This course was justified because of the thorough studies, surveys, and sub-surface investigations which preceded the issuing of the contract documents, and because the complete and detailed drawings and specifications reduced to a minimum the uncertainties faced by the contractor.

The decisions of the Chief Engineer were, by the contract agreement, final on all questions relating to the work and its performance, and the contracts provided that the contractor must proceed with the work in all events. However, since the contractor was limited to money damage only, it provided that should he object to any decision of the Chief Engineer in regard to compensation, such disputes should be referred to arbitration under the rules of the Committee on Arbitration of the Chamber of Commerce of the State of New York. It was required that arbitrators should be engineers experienced in matters of the nature covered by the contract, selected from those listed as "Engineers" or "Consulting Engineers" in the Chamber of Commerce list of Official Arbitrators. They were empowered to make decisions as to questions of fact, which, in turn, could be submitted to appropriate Courts as a basis for the determination of legal rights and liabilities, including questions of the reasonableness of the Chief Engineer's decisions.

Wherever possible, latitude was given to bidders for applying their ingenuity, experience, and facilities toward developing the most economical methods for performing the work. The contract for the foundation piers for the New Jersey towers provided for competitive bidding on two types of construction; namely, pneumatic caissons or open coffer-dams. It had been found impossible to determine in advance which of these methods would be most economical. Therefore, complete drawings, specifications, and schedules of prices were provided because both methods and bidders were permitted to submit tenders on either or both methods, selection to be reserved until after the receipt of bids. The lowest bid for the coffer-dam method was considerably less than the lowest bid on the caisson method, and, therefore, the contract was let for the former. Provision was made, however, that if a bidder submitted tenders on both methods, the prices quoted on the one not adopted formed no part of the contract, so that, after the contract had been awarded, should the contractor decide to revert to the other and more expensive method, he would be obliged to perform the work under the prices quoted for the cheaper one. If, however, after the award of the contract it became advisable, in the

opinion of the Chief Engineer, to revert to the more expensive method, he had the authority to apply the unit prices of either method. This provision foreclosed the possibility of a contractor wilfully submitting a low bid on one method with the idea that, after he had been awarded the contract, he could revert to the other at a cost to the Port Authority that might possibly be higher than the low bid on that method.

For several reasons unusual contract provisions were necessary in connection with the steelwork of the main bridge (the cables, the towers, and the floor steel): (*a*) It was deemed advisable not to determine in advance of receiving bids whether all this work should be handled as one contract or whether the work should be separated into two contracts; and (*b*) it was thought desirable to call for bids on two alternate types of cables—parallel wire or eye-bar—and also on alternate methods of procedure and arrangement. By one method of procedure all four cables would be constructed simultaneously, while by the alternate method one cable of each pair would be constructed first and the second subsequently while the floor steel was being erected. As for arrangement, bids could be made upon the basis of the two cables of each pair constructed side by side or one above the other.

Decision as to contracting for this work as a whole or in two parts could not be made in advance because it was felt that some contractors might wish to submit a more favorable tender for the entire work than they would if proposals were received separately for the cables and for the towers and floor steel, and these separate bids simply added together. In order to provide for these various alternate arrangements and types of cables, complete contract drawings and specifications were drawn, covering both types of cables, and six separate proposals were provided to cover the various conditions.

The contract for this entire work was designated "HRB-5" that for the towers and floor steel, "HRB-5*A*," and that for the cables, suspenders, and anchorage steelwork, "HRB-5*B*" (see Table 1). Contractors desiring to bid on both parts as a unit were instructed to make their bids on the proposals for Contract HRB-5; contractors desiring to bid on both parts of the structure separately were instructed to fill in the proposal blanks on Contracts HRB-5*A* and HRB-5*B*; and the contract provided that the acceptance of either one of these proposals would constitute a rejection of the other proposal. Finally, bidders desiring to bid on only one part of the work were instructed to fill in the proposal blank for Contract HRB-5*A* or Contract HRB-5*B*, as the case might be. In order to provide against the contingency of receiving a favorable quotation on one type of cable, and not receiving a suitable proposal for the towers for that type, contractors submitting proposals on Contract HRB-5*A* were required to bid on the designs for both the eye-bar and wire cables.

Due to the acute competition at the time between wire-cable interests and eye-bar cable interests, and the comparatively few contractors qualified to bid on this part of the work, there was the further possibility that contractors qualified to bid on the entire structure would do so without bidding separately on the towers and floor steel. This would result in a lack of competition on Contract HRB-5*A* for the towers and floor steel

as a separate unit. It was known that at least two or three concerns were interested in wire cables; but it was expected that they would not submit proposals for the towers and floor steel, and, therefore, it was possible that, because of failure to obtain a satisfactory proposal for the towers and floor steel alone, it might be impossible to take advantage of a low proposal on the cables. To prevent these contingencies, it was further required that, inasmuch as the submission of a proposal for the entire steelwork indicated a willingness on the part of the bidder to construct the towers and floor steel, either directly or through a sub-contractor, every bidder submitting a proposal for the entire steelwork was required also to submit a proposal for the towers and floor steel alone, or to arrange to have his proposed sub-contractor submit such a bid.

On October 3, 1927, proposals were received from five bidders, resulting in fourteen different combinations covering the steelwork. The lowest bids were accepted, decision being made thereby upon the various alternates, as follows:

(1) Wire cables were adopted in preference to eye-bar cables.
(2) The cables were to be placed in pairs, side by side, instead of one above the other.
(3) One cable of each pair was to be constructed first, so that erection of the floor steel could proceed at the earliest date, while the other two cables were being constructed.
(4) The steelwork was let to two different contractors, one for the cables, suspenders, and anchorage steelwork, and the other for the towers and floor steel.

Further studies made by the contractor for the cables subsequent to the letting of the contract developed a material advantage, both to the contractor and to the Port Authority, in the construction of all four cables simultaneously and, accordingly, although the contract had been awarded on the basis of successive erection of the cables, an agreement was made in the spring of 1929 whereby the cable contractor was to erect all four cables simultaneously under conditions such that the date for completion of the entire work, and incidentally that for completion of the entire structure, was materially advanced.

Specifications.—In the preparation of the specifications, the same care was given as in the preparation of the Form of Contract and the contract drawings, with the intent of insuring work of the highest quality in every respect. It was realized that the specifications were to serve as instructions to the bidder as to just what work was to be performed, and as to the standard by which the quality of work was to be controlled. To accomplish these purposes, it was realized that the specifications had to be complete, definite, and clear.

Each set of specifications was arranged in a series of chapters, so as to place the requirements before prospective bidders clearly and conveniently, and so as to provide for easy reference in the future by the contractor and the engineers. The particular chapter divisions adopted for any contract depended entirely upon the nature of the work to be performed under that contract, and no "standard" arrangement was considered desirable. An earnest effort was made continually not to fall into the error so frequently

made in the preparation of specifications, of blindly copying provisions made in a previous specification without carefully considering whether the provision applied properly or whether modifications were required to meet the particular conditions.

The practice was to have as the first section a chapter entitled "General Provisions." This was invariably begun with a carefully prepared statement of the work covered by the specifications. The statement was intended in no way as a limitation upon the work to be performed, since, by the terms of the contract, the contractor was obligated to do everything necessary or proper for, or incidental to, the completion of the particular part of the work covered by that contract. It was intended rather as a comprehensive picture of what was contemplated under the contract, so that the contractor could read the entire specifications with a better understanding. This statement was followed by the specific requirements of a general nature, such as the co-operation with contractors engaged in other parts of the construction work; precautions to be taken to safeguard against injury or damage to the work itself, to traffic, or to persons or property; temporary structures to be required for the work, such as fences, falsework, walkways, etc.; sanitary provisions to be required; and like general provisions.

The parts of the specifications that followed the "General Provisions" were subdivided into convenient chapters. In case the work involved many trades, as, for example, in a contract for a building, the chapters were separated into distinct classes of work which the general contractor might wish to sub-contract. If the work involved only one, or only a few types, of construction, however, the chapter divisions were based on the different classes of workmanship or construction involved. In a contract for fabrication and erection of structural steel, for example, separate chapters were devoted to design and detail, for the use of the drafting-room; to manufacture and workmanship, for convenience of the shop; to field work, for the requirements in erection; to materials, for the benefit of the mills; and, finally, to inspection and tests. As far as they were applicable, reference was made to the Standard Specifications of the American Society for Testing Materials, and, with the consent of that Society, reprints of the specifications referred to were bound with the printed contract papers, except where the reference was of only secondary importance.

Special care was given to avoid repetition, and to use phraseology that was plain, simple, straightforward, definite, and free from ambiguity. It was realized that the specifications, even more than the other parts of the contract, are read by different types of people—the engineer, the contractor, the material men, and, when disputes arise, the lawyers and the Courts—and that carefully chosen wording in the specifications, therefore, was essential. In general, because of the fact that the contractor was held responsible, under the terms of the contract, for all damages arising out of the work, the effort was made to specify the results desired, and not to specify methods by which the work should be performed, in order to avoid any implication, in case of damages, that the contractor was merely following instructions and that, therefore, the engineer shared the responsibility with him.

It was further borne in mind that repetition can easily lead to ambiguity and inconsistency and that, under such conditions, the Courts invariably rule that the contractor is entitled to the interpretation most favorable to his interests. In certain cases, it did become absolutely necessary to repeat, but in all such cases care was exercised to see that the two statements were consistent.

Throughout all the work, the Port Authority consistently refrained from specifying a proprietary article of a trade brand or of a particular manufacture. It sometimes happened, however, that certain materials were wanted the manufacture of which had become so standardized that it was literally impossible, except by going into disproportionately great detail and length, to indicate what was desired. In such cases, the article of specific brand or manufacture was clearly specified as a standard by which competing materials of the same nature were measured, and the good old time-worn phrase, "or an approved equal," was inserted.

Contractors

The successful bidders on work listed in Table 1 are: Contract HRB-2, Silas B. Mason, Incorporated; Contract HRB-3, Foley Brothers, Incorporated; Contract HRB-4, Arthur McMullen Company; Contract HRB-5*A*, McClintic-Marshall Company; Contract HRB-5*B*, John A. Roebling's Sons Company; Contracts HRB-6 and HRB-8, Cornell Contracting Corporation; Contract HRB-7, Klosk Contracting Company; Contract HRB-9, William P. McGarry Company; Contracts HRB-10 and HRB-11, George M. Brewster and Son, Incorporated; Contract HRB-12, Corbetta Concrete Corporation; Contract HRB-13*A*, Robert J. Murphy, Incorporated; Contract HRB-13*B*, Andrew I. Green, Incorporated; Contract HRB-13*C*, Hoffman-Elias, Incorporated; Contract HRB-13*D*, John Boyd Plumbing and Heating Company; Contract HRB-14, Beach Electric Company, Incorporated; Contract HRB-15, De Riso Construction Company; Contract HRB-16, Skolnick Building Corporation; Contract HRB-17, Salkind Company, Incorporated; and, Contract HRB-18, Auf der Heide Contracting Company.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

FOUNDATION TREATMENT AT RODRIGUEZ DAM[1]

By Charles P. Williams,[2] M. Am. Soc. C. E.

Synopsis

This is an account of the design and construction of the foundation structure for the Rodriguez Dam, on the Tijuana River, Baja California, Mexico. The dam is of the Ambursen type, and, as designed, has a greater height than any dam of that type heretofore built. The bed-rock in parts of the stream bed is of inferior quality, and, in some parts, disintegrated. A geologic fault about 20 ft. in width extends along the border of the stream bed.

A brief introductory description of the project and of the dam is given, the geologic formation at the site is described, and the foundation structure that supports that part of the dam which otherwise would rest on poor foundation, is described and illustrated.

Introduction

The Rodriguez Dam is situated on the Tijuana River, in the northern part of Baja California, Mexico, about 11 miles easterly from the Town of Tijuana. Its purpose is the storage and diversion of water for the irrigation of about 5 000 acres in the Tijuana Valley in Mexico, and for domestic supply for the municipality of Tijuana which has a population of 10 000.

The drainage basin of the river, which lies partly in Baja California and partly in San Diego County, California, has an area of 1 670 sq. miles. This is greater than that of any other stream south of Los Angeles, Calif., and west of the Colorado River; it nearly equals the total area of the drainage basins of all other streams in San Diego County. The stream has two principal branches, upon the larger and more southerly of which the Rodriguez Dam has been located. The area of the drainage basin above the Rodriguez site is 938 sq. miles.

Note.—Discussion on this paper will be closed in January, 1933, *Proceedings*.

[1] Presented at the Meeting of the Irrigation Division, Sacramento, Calif., April 24, 1930.

[2] Cons. Engr., Los Angeles, Calif.

Like all streams in the vicinity, the flow of the Tijuana River in ordinary years is very small, and the supply is necessarily dependent upon the storage of flood waters. Periods of from 5 to 7 years, in which the flow is insignificant, may be expected, and consequently large carry-over storage is important. In order to furnish a sufficient irrigation and municipal supply from Rodriguez Reservoir, a capacity of about 110 000 acre-ft. is necessary, requiring a maximum reservoir water surface about 180 ft. above the stream bed.

While streams in this vicinity, even during the rainy season, have very small flows in ordinary years, excessive floods sometimes occur. The largest flood of definite record was that of 1916. Some of the streams in San Diego County, at the peak of that flood, had flows corresponding to coefficients ranging from 5 to 7 in Talbot's formula. At that time, the flow of the Tijuana River, below the Arroyo Matanuco, which empties into the river immediately below the Rodriguez Dam site, and which has a drainage area of only 115 sq. miles, was about 70 000 sec-ft. It is estimated, however, that a flood of 150 000 sec-ft. is possible.

The Rodriguez site is in a gorge which, at its narrowest section, has a width at stream bed of about 100 ft.; at 130 ft. above stream bed, the width is about 750 ft. Beyond this, on either side, the gorge is flanked by a saddle and by gentle slopes which, immediately adjacent to the site, rise to 200 ft., or more, above stream bed.

Rock outcrops throughout a considerable portion of the site, the rock of the canyon walls at the gorge, that of the western slope, and that of a part of the eastern slope being rhyolite, while that of the easterly part of the eastern slope is granite. The rock is fused at the contact between the granite and the rhyolite, showing the former to be the older rock, and indicating that no great leakage from the reservoir may be expected along the contact. The surface rock of the canyon walls, in the narrowest section, is fresh and hard, although considerably broken; that of the slopes is generally badly weathered, and, in some parts, considerably disintegrated. On either side of the canyon, are a number of cleavage planes which dip toward the stream bed and, in some cases, up stream.

Preliminary Investigations

In the preliminary investigation of the dam site, nine borings were made in the stream bed and fifty-two test pits were excavated on the side hills. The latter indicated that suitable rock for foundation could be secured, ordinarily, with comparatively shallow excavation. At the site of the east wing of the dam, for a distance of about 500 ft., suitable rock was found at depths ranging from 15 to 25 ft., while at the site of the proposed spillway, on the west side of the river, the depth for about 100 ft. was from 15 to 40 ft. Five of the nine borings in the stream bed indicated satisfactory rock at depths not exceeding 50 ft. From four of the borings, no cores could be obtained, the borings being made easily with a chopping bit. The materials obtained from two of these four borings were clean sharp stone chips, but the materials from the other two gave considerable evidence of disintegration.

After the borings had been made and a number of the test pits had been excavated, the geological formation at the site and in the vicinity, both in

Mexico and in the United States, was examined by a geologist, who reported that, in his opinion, the rock in the stream bed, that of the western flank, and that of a part of the eastern flank, would be found suitable for a foundation for a concrete dam, but that he considered an earth and rock-fill dam to be best suited for the saddle on the eastern flank.

Brief Description of the Dam

The dam is of the Ambursen type. Its maximum planned height is 187 ft. above the stream bed and about 240 ft. above the lowest foundation bed-rock. The length of the crest is about 2 000 ft. For the central part of the dam and for the west wing, the spacing of the buttresses is 22 ft., center to center. The buttresses for the central part and the west wing vary in thickness from 19 in. at their thinnest section, to 66 in. at 200 ft. below the crest. The thickness of the deck varies from 25 in. at 15 ft. below the crest to 64.5 in. at its lowest portion, 192 ft. below the crest. The lower face of the deck has a slope of 1 on 1, and the down-stream face of the buttresses a batter of 5 on 1.

In each of three of the stream-bed bays of the dam, the plans provide for a 5 by 5-ft. sliding sluice-gate, operated by a hydraulic cylinder. The service outlet works consist of two 30-in. cast-iron pipes, at an elevation about 33 ft. above the stream bed, in each of which will be installed one 30 by 24-in. needle-valve, and two 30-in. emergency gate-valves. The discharge will be measured by a Venturi meter.

The spillway in the west wing of the dam is to be controlled by nine 30 by 30-ft. structural steel gates on caterpillar bearings. With an elevation of the water surface 6.5 ft. below the crest of the dam, the spillway is designed for an estimated capacity of 150 000 sec-ft.

The Foundation

On February 17, 1928, Gen. Abelardo L. Rodriguez, Governor of the Northern District of Baja California, entered into a contract with the Ambursen Dam Company for the design and construction of a dam at the Rodriguez site, to be subject to the approval of the Government.

Excavation in the stream bed was begun by sub-contractors on May 21, 1928. In August, bed-rock was encountered at several points near the up-stream limit of the excavation, at about 35 ft. below the original elevation of the stream bed. The rock was much broken and not suitable for buttress foundations. A geologic fault, about 20 ft. in width in the exposed portion, which extended in a direction nearly parallel to the stream bed along its eastern margin, and which had not been indicated by surface conditions, was revealed by the excavation. The excavation for the cut-off wall, begun in August, disclosed badly disintegrated rock in mid-stream.

Early in September, Dr. F. L. Ransome, Consulting Geologist, Dr. Paul Waitz, Consulting Geologist for the National Commission of Irrigation, of Mexico, and the late A. J. Wiley, M. Am. Soc. C. E., were called for consultation regarding the conditions affecting the suitability of the site for

the construction of a dam of the type proposed. The problems to be considered were the possibility of excessive leakage from the reservoir, the practicability of obtaining adequate foundation for the dam, the probable depth and extent of cut-off wall required, and the possible danger of future seismic

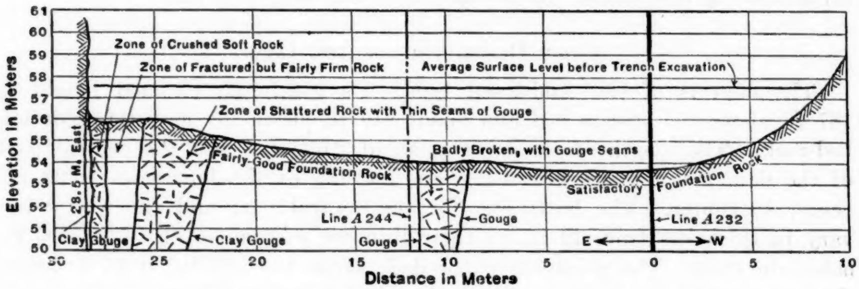

FIG. 1.—RODRIGUEZ DAM: LONGITUDINAL SECTION OF CUT-OFF TRENCH IN STREAM BED.

disturbance. A further inspection was made by Messrs. Wiley and Ransome on November 11 to 13, 1928, at which time the cut-off trench had been excavated to a depth of about 33 ft. below the bed-rock surface.

After this inspection, Dr. Ransome reiterated his opinion as reported previously under date of September 13, 1928. In the report mentioned it was pointed out that the site chosen was satisfactory in most respects. One grave defect, however, was the existence of a fault zone which, although it had been

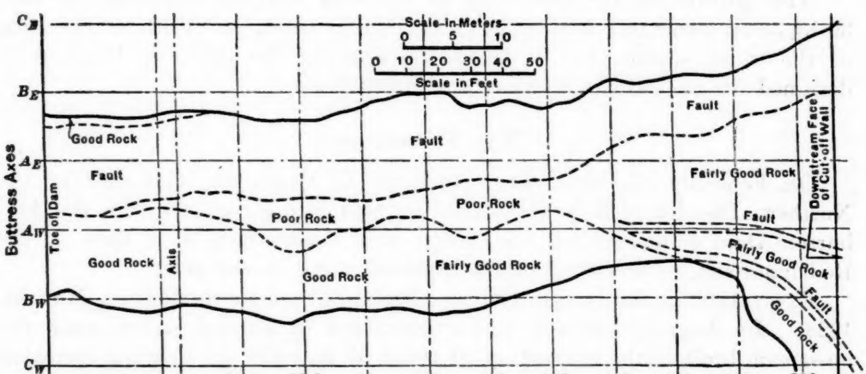

FIG. 2.—CHARACTER OF FOUNDATION ROCK IN RIVER CHANNEL.

inactive for more than 100 years, might conceivably move in the future. Nevertheless, such movement is not expected and Dr. Ransome stated that the risk was not so great as to justify the condemnation of the site.

A longitudinal section of the cut-off trench in the stream bed, showing the formation disclosed at the time of Dr. Ransome's inspection, is shown by Fig. 1. The general character of the rock in the stream bed is shown by Fig. 2.

The bearing power of the rock in the stream bed varies greatly in different parts of the foundation area. Four of the buttresses, if founded on the natural material, would rest for a part of their lengths upon material of relatively poor bearing power. The easterly two of the four would be founded for part of their lengths upon the material of the main fault. It was realized, therefore, that it would be necessary to support the four buttresses by a structure of a type such that the loads from the weaker parts of the foundation area would be transferred to those parts having ample supporting power.

After a number of preliminary studies had been made a design was submitted providing for a reinforced concrete floor, 8 ft. thick, covering the entire stream bed within the foundation area. Above, and tied to the floor by steel reinforcement, were massive reinforced concrete ribs, 5 ft. thick and spaced 20 ft., center to center. These ribs were designed so as to transfer the buttress loads, by internal arch action, to the sound rock on either side of the stream bed. The internal arches were defined within the ribs by means of bands of reinforcing steel which were extended through the buttresses, the buttresses and ribs forming a cellular structure. Besides serving as a support for the buttresses, the ribs were intended to serve as horizontal bracing for the rock of the canyon walls, which was cut by a number of cleavage planes sloping toward the stream bed.

The design was reviewed by Fred A. Noetzli, M. Am. Soc. C. E., and was found sufficient for the purpose for which it was intended. Later, it was considered by a Consulting Board consisting of Messrs. Wiley, Noetzli, and the writer, assisted by Spencer W. Stewart, M. Am. Soc. C. E., which Board concluded that while the proposed structure had been found by analysis to be reasonably sufficient, a more satisfactory and only a little more costly structure would be obtained by building a voussoir concrete arch, continuous from the up-stream cut-off wall to the toe of the dam, with a curved extrados and a flat base, resting on the natural material. It was proposed that the arch consist of unreinforced massive voussoir blocks with staggered radial joints, each block extending, without construction joints, from the base to the extrados.

In designing the structure the upper part was to be considered an arch, with the theoretical intrados intersecting the canyon walls above the lower limit of sound rock. It was to be assumed that the load of the concrete below the theoretical intrados would be carried by the natural foundation material, and that the arch would support only the buttress loading with reservoir water surface at the elevation of assumed high water, the weight of the arch above the theoretical intrados, and the water load between the buttresses to the elevation of assumed low water below the dam. Since the structure was assumed to be fully submerged at all times, stresses due to variations in temperature were to be neglected. Uplift under the structure, due to low-water pressure, was to be assumed. Further consideration of the problem led to the adoption of a definite construction joint at the intrados.

On account of the importance of the problem, independent designs were prepared by Mr. Noetzli and the contractor. These were considered by a

board consisting of Messrs. Wiley and Noetzli, C. V. Davis, Assoc. M. Am. Soc. C. E., and the writer, and final conclusions were reached regarding the type of structure and important details.

The essential features of the adopted design are shown by Figs. 3 and 4. From Fig. 3 it will be observed that the four buttresses, *B* East, *A* East, *A* West, and *B* West, are supported by the arch. In order, in so far as possible, to avoid undesirable twisting moments in the arch barrel and irregular deflection along the line of support of any buttress, the axial line

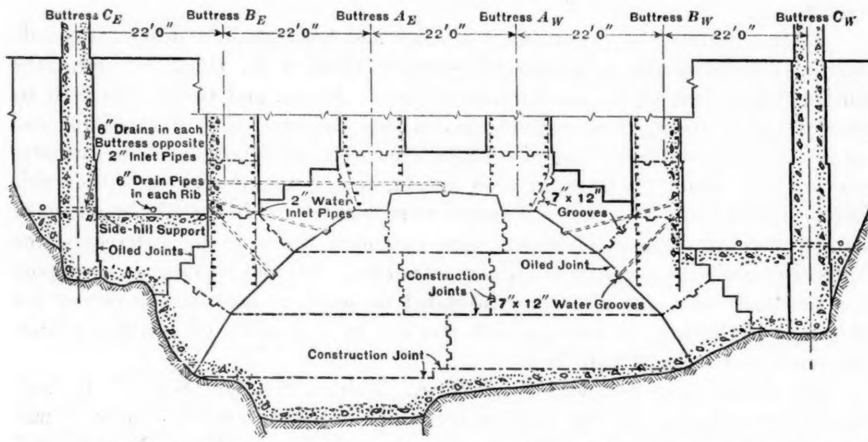

FIG. 3.—TYPICAL SECTION OF FINAL DESIGN, SHOWING TRANSITION, RODRIGUEZ DAM.

of the arch crown was made parallel to and equi-distant from the center lines of the buttresses, *A* East and *A* West. This made it necessary, at some sections, to use arch spans of greater length than would have been required to span the fault and the rock of inferior quality. On account of the varying widths to be spanned, and of the non-uniform loading, it was necessary to use arches of greater rise at the up-stream and down-stream parts of the structure, than in the central part, as shown by Fig. 4.

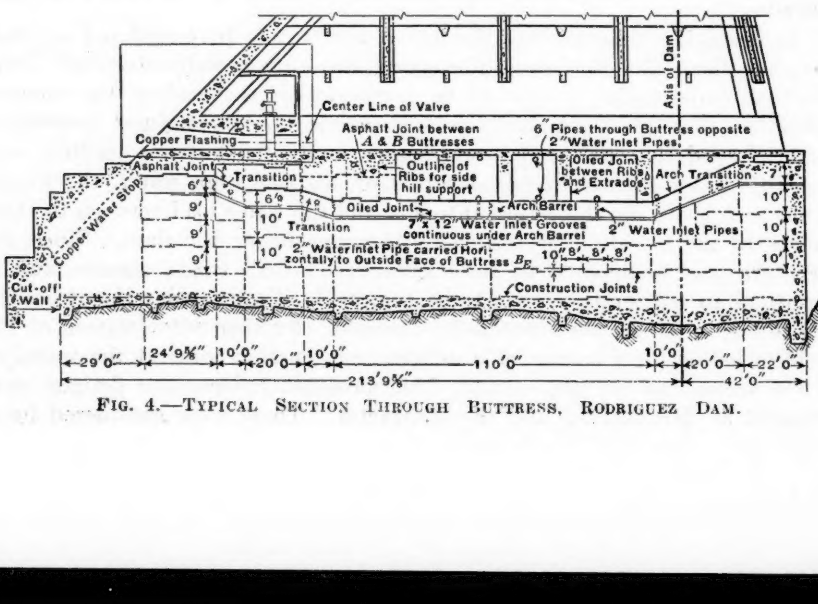

FIG. 4.—TYPICAL SECTION THROUGH BUTTRESS, RODRIGUEZ DAM.

The arch is designed to carry the full superimposed load. It is expected, however, that a considerable part of the load will be supported through the sub-intradosal concrete by the natural material in the stream bed. Where, however, such material has not supporting power sufficient for the entire load and there is incipient settlement, a part of the load will be taken by the arch, thus relieving the natural material of as much of this load as may be necessary to prevent further settlement.

Before beginning the construction of the sub-intradosal concrete blocks, cut-off trenches about 3 ft. deep, 2 ft. wide, and spaced 25 ft., center to center, were cut across the stream bed, in the fault zone, and where the rock was of inferior quality. The stream bed was then covered with a concrete mat, not less than 2 ft. in thickness. Substantial key-ways, to prevent sliding, were constructed in the upper surface of the mat.

The sub-intradosal concrete was constructed of inter-locking blocks, as shown in Fig. 3. The upper surfaces of the blocks, upon which the arch barrel rests, were finished smooth and painted with oil to prevent adhesion. Transverse keys about 6 in. deep and 10 ft. wide, were constructed in the sub-intradosal blocks in order to prevent sliding of one course upon another, or sliding of the arch upon the blocks.

Normally, the water pressure under the arch will be that due to low water below the dam. During a flood, the water below the dam might rise within a short time to 15 ft., or more, above low water. This, coming at a time when the reservoir would be full, would impose a heavy load upon the arch, unless provision was made for equalizing the pressures upon the extrados and the intrados. To accomplish this, water-inlet grooves were constructed the entire length of the arch, with 2-in. inlet pipes at 15-ft. and 20-ft. intervals. It was first intended that these grooves should be filled with screened gravel, ranging in size from $\frac{1}{4}$ in. to $\frac{3}{4}$ in., the gravel to be covered by a layer of porous mortar. Owing to some of the grooves being on steeply sloping surfaces, it was found very difficult to retain this type of filling in place. The grooves were filled, therefore, with pre-cast blocks of mortar, consisting of 1 part cement to 10 parts of coarse manufactured sand. These blocks were so porous as to offer very little resistance to the passage of water. After being placed in the grooves, they were covered with a coat of mortar consisting of 1 part cement and 3 parts river sand.

Between the haunches of the arch and the walls of the canyon, concrete filling was placed to varying heights, depending upon the apparent stability of the rock walls. Above this filling and above the extrados of the arch, concrete walls 5 ft. thick, spaced generally 20 ft., center to center, have been constructed to the elevation of the stream bed, these walls, as well as the haunch filling, being for the purpose of bracing the side walls of the canyon, as a preventive of possible sliding on planes of cleavage.

In order that it may be possible at any time to determine the upward water pressure at various points at the base of the foundation structure, thirty-five 4-in., galvanized and dipped pipes have been placed in the structure, leading from the points at which the uplift pressure is to be measured,

a few feet upward, then horizontally to four centralized points, from which they continue vertically upward through the structure. Eventually, the pipes will be extended vertically upward to observation platforms above high water below the dam. The lower end of each vertical riser will be always below the hydraulic grade at the point where the pressure is measured. The pressure is determined by observing the elevation of the water surface in the riser pipes, or by a pressure gauge, if, in any case, the elevation of the hydraulic grade at the point to be measured is higher than the top of the riser pipe. The plan of the pipe system is shown by Fig. 5.

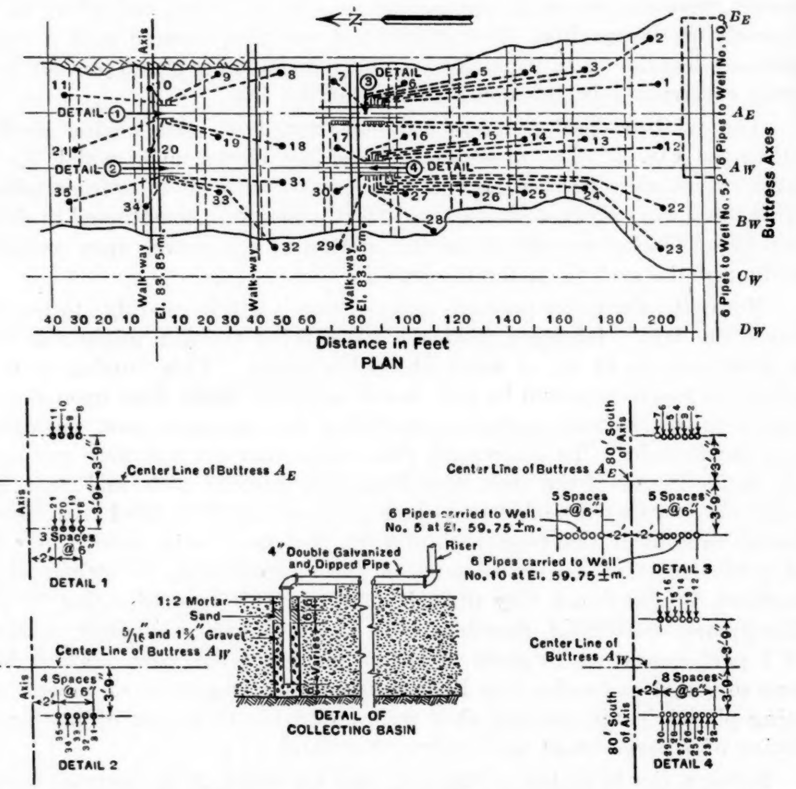

FIG. 5.—PLAN SHOWING LOCATION OF RISER PIPES FOR FOUNDATION TESTING SYSTEM, RODRIGUEZ DAM.

The arch barrel resembles a frame type of structure, which was adopted as the most suitable because of the large concentrated buttress loads. The stresses were computed by the elastic theory. The procedure followed in designing the several theoretical sections of the arch barrel was, as follows: Having determined the thickness and other dimensions of the arch rib by trial computations at a given section of the arch barrel (on the basis of its span and rise), the pressure line for the loads and weights was drawn and the arch axis made to coincide with this pressure line. The direct com-

pression stresses at the crown and at the abutment could then readily be computed. The stresses due to rib-shortening, shear, and temperature variations were then calculated by the method of least work. These stresses, added algebraically to the direct compression stresses, gave the maximum and minimum stresses at the crown and at the abutments. The half section of the arch was divided into ten hypothetical voussoirs, and the stresses were computed at each voussoir point.

The working loads adopted for the arch-barrel designs were, for maximum compression, 700 lb. per sq. in., and for maximum tension, 50 lb. per sq. in. The final designs showed no tension in any arch section and no reinforcement was used.

The intended minimum compressive strength of concrete at 28 days, adopted for the mat, for the arch barrel, and for the 5-ft. bracing walls, was 2 000 lb. per sq. in., and that for the sub-intradosal concrete and the haunch filling, 1 200 lb. per sq. in. At the beginning of construction, the volumes of materials used per batch, in making concrete of the intended strength of 2 000 lb. per sq. in. at 28 days, were as follows:

Cement		8 sacks
Sand		18.3 cu. ft.
Gravel:		
$\frac{5}{16}$ to $1\frac{3}{4}$-in.		13.8 cu. ft.
$1\frac{3}{4}$ to $3\frac{1}{2}$-in.	13.8 cu. ft.	or 24.0 cu. ft.
$3\frac{1}{2}$ to 6-in.	13.8 cu. ft.	
Water-cement ratio		1.0

Afterward, it became evident that the desired strength could be obtained with a greater water-cement ratio, and the ratio was increased to 1.08. The use of the greater ratio made it possible to reduce the quantity of cement per batch to 7 sacks without reducing the workability appreciably. In the concrete for the sub-intradosal blocks, the volumes of materials per batch first used were: 6 sacks of cement and a water-cement ratio of 1.25. The sand and gravel quantities were the same as for the 2 000-lb. concrete. The number of sacks of cement per batch was reduced afterward to $5\frac{1}{2}$, with no change in the water-cement ratio.

The design of the transition structure between the flexible arch barrel and the rigid cut-off wall was a problem of some difficulty. The transition is shown in cross-section by Fig. 3. It consists of a massive block of concrete having a vertical depth ranging from 40 to 50 ft. and spanning the entire stream bed. The block overlaps the cut-off wall and is tied by steel reinforcement to the arch barrel. The bottom is reinforced longitudinally with three layers of $1\frac{1}{2}$-in. square steel bars, spaced 12 in., center to center, to resist possible tension due to bending in areas where the natural foundation has insufficient supporting power. It is reinforced at its easterly end by seven courses of $1\frac{1}{8}$-in. square diagonal tension bars, spaced 24 in., center to center, to aid in carrying the load at the fault to the solid rock on the eastern bank. It is also reinforced for possible diagonal tension due to its support by the cut-off wall.

Cut-Off Wall

When the fault in the stream bed was discovered, it was realized that it would be necessary to carry the cut-off wall at the fault to a great depth. There was considerable doubt, however, regarding the depth to which it would be necessary to construct the wall in the western portion of the stream bed. Excavation of the cut-off trench was begun in open cut and was carried by this method to a depth of from 26 to 33 ft. below the original rock surface, corresponding to depths of from 66 to 72 ft. below the stream bed. The greater part of the trench required timbering. When the depths mentioned had been reached, maintaining the open cut had become so difficult that a

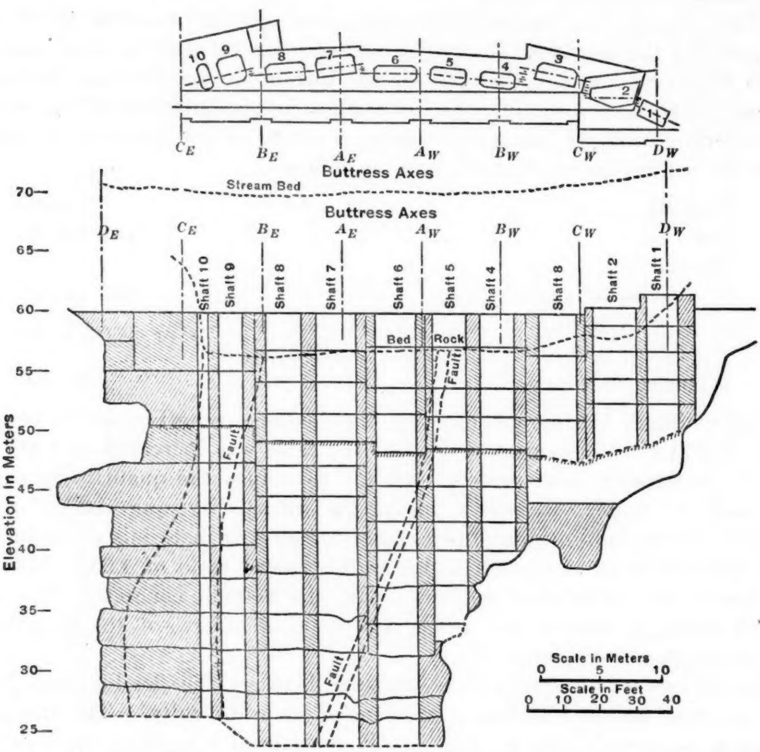

Fig. 6.—Cut-Off Wall, Plan and Longitudinal Section at River Channel, Rodriguez Dam.

change of method was necessary, and it was decided to continue the excavation through shafts. The trench was filled with concrete, in which ten shafts were constructed, through which the excavation and the placing of concrete were continued. The shafts are shown in plan and section by Fig. 6. The walls of the shafts were reinforced throughout the zone of inferior rock and well into the solid rock on either side. Stop-water key-ways were constructed in the walls, and, when excavation shall have been completed, the shafts will be filled with concrete or puddle, as then may be considered desirable.

Numerous grout pipes were inserted in the shaft walls, where the rock was broken or seamy, and two lines of grout holes, staggered in two rows 5 ft. apart, and spaced not exceeding 10 ft. apart in each row, were drilled from the bottoms of the shafts throughout that part of the length of the wall, where final depth has been reached. It is planned to drill numerous grout holes adjacent to and eastward from the fault, along the line of the cut-off wall extended, where the rock, although hard, is more or less broken.

Provision was made for the installation of twelve pressure pipes for determining the hydraulic gradient at various points on the up-stream and down-stream faces of the cut-off wall.

The length of wall constructed through shafts was, at the beginning, about 150 ft. At a depth of about 66 ft. below the original rock surface, the length had decreased to about 110 ft. Excavation and concrete shaft-walls have been carried to a depth of about 112 ft. below the original rock surface, or 150 ft. below stream bed, at which depth the length is about 90 ft. It is intended to carry the wall at the fault, and as far on either side as may be found necessary to secure a key in suitable rock, to a depth of 300 ft. below stream bed.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

LAKE CHAMPLAIN BRIDGE

Discussion

BY CHARLES M. SPOFFORD, M. AM. SOC. C. E.

CHARLES M. SPOFFORD,[11] M. AM. SOC. C. E. (by letter).[11a]—The writer has been gratified by the favorable comments on this paper, and takes this opportunity to express his thanks for such comments.

As stated by Mr. Abbett the method of deflections might have been used advantageously in computing the stresses in the trusses. Many years ago, in connection with an investigation made in behalf of the Comptroller of the City of New York, the writer used the method of deflections in computing the stresses in all the members of the Queensboro Bridge. This is a continuous structure statically indeterminate to the second degree (as is the Lake Champlain Bridge). Later experience in dealing with statically indeterminate structures has convinced him that the method of least work is simpler to apply, largely because the signs of the various expressions take care of themselves, requiring the computer to give attention to the numerical computations only. As a matter of fact, both methods are identical when reduced to their simplest terms.

As far as graphical methods go the writer's experience indicates that analytical methods are more satisfactory in cases where the computations may be divided among several staff members. One of the chief advantages of an analytical method is the fact that the results may be checked with more precision than by any graphical method known to the writer, thus giving a greater feeling of security than would otherwise be the case. At all events the computations form such a small factor as far as the expense of designing goes that any theoretically correct method may be applied without affecting materially the expense or time required for the design.

With respect to laitance in the concrete piers, the concrete was properly proportioned, thoroughly mixed, and carefully and continuously placed in position by bottom dump 1-yd. buckets. As the pouring progressed the upper

NOTE.—The paper by Charles M. Spofford, M. Am. Soc. C. E., was presented at the meeting of the Structural Division, Boston, Mass., October 10, 1929, and published in December, 1931, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1932, by Messrs. Robert W. Abbett, Henry W. Troelsch, Jacob Feld, and Clarence W. Hudson.

[11] Hayward Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

[11a] Received by the Secretary September 10, 1932.

surfaces of the piers were inspected by divers from time to time, who reported little or no laitance. At the conclusion of pouring the amount of laitance that had to be removed was a matter of only 2 or 3 in. instead of the 2 ft. suggested as a possibility by Mr. Feld. Whether the strength of the concrete was reduced by the formation of laitance during pouring was not investigated, but tests of the concrete as it came from the mixer showed it to be of high strength; and as the stresses to which it was subjected were not severe, no doubt existed as to its having adequate strength. Inspection of the character of concrete deposited in sea water in the piers of the Charlestown Bridge, in Boston, Mass., were made some time ago, after the concrete had been in place for several years. Samples were taken by diamond-drill borings, under the direction of Frederic H. Fay, M. Am. Soc. C. E. This inspection showed the concrete to be of excellent quality throughout the depth of the pier.

In reply to the query by Mr. Feld as to the reason for not protecting the pier concrete against abrasion and deterioration, it may be pointed out that the piers are in a fresh-water lake with little current and are practically free from danger of abrasion from ice and floating objects. The concrete piers of the Queen City Bridge, across the Merrimac River (designed and constructed in 1922 at Manchester, N. H., under the supervision of the writer's firm), which are located in fresh water and a short distance below a dam, have undergone no deterioration, either from the action of the current or from other causes, as far as the writer knows. Had the Lake Champlain Bridge piers been built in tidal water, granite encasement between high and low water would have been used in accordance with the writer's usual practice.

As to the relative cost of piers constructed in open caissons as compared with those built by the pneumatic process, the writer can only judge by the actual bids received, which were mentioned in the paper. The specifications stipulated that the design of the caissons should be left to the contractor who might use timber, steel, or concrete, and presumably some of these contractors bid on caissons designed with either pre-cast concrete or steel shells, if they found such a construction economical as suggested by Mr. Feld.

In reply to Mr. Hudson's comment on the longitudinal gradient, it should be borne in mind that the bridge is in a country where steep hills are frequent and where the $5\frac{1}{2}\%$ gradient used in the bridge is moderate and substantially less than that on some of the main highways in the vicinity. No difficulties from the gradient have yet been reported to the engineers.

As stated in the paper, the bridge was designed for the *H*-15 loading given in the Standard Specifications for Steel Highway Bridges of the U. S. Department of Agriculture, with the exception that 20-ton trucks were used instead of the 15-ton trucks specified in the loading. The engineers believe that, in view of the probable infrequency and possible non-appearance of such trucks on the bridge, it was entirely logical to apply a higher unit loading for the floor system than had the 15-ton trucks conforming to the *H*-15 loading been used.

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DISCUSSIONS

CONSTRUCTION PLANT AND METHODS FOR ERECTING STEEL BRIDGES

Discussion

BY MESSRS. A. A. EREMIN, AND A. F. REICHMANN

A. A. EREMIN,[7] ASSOC. M. AM. SOC. C. E. (by letter).[7a]—With the advancement in modern bridge design considerable attention is generally given to the problem of erection. Often the equipment available and the economic considerations of the erecting methods decide the selection of the type of bridge.

An interesting type of semi-permanent bridge with a half-through truss of uniform depth was developed by M. Pigeaud,[8] for the French Army, during the World War, which deserves careful consideration of military engineers.

Parallel cables are swung over some steel-framed square towers, which are composed of members that can easily be handled. Cross-beams are suspended from the cables, and tracks are placed on top of the beams. The trusses are made in sections about 15 ft. long which are rolled into position in the span by means of a car. After completing all connections the entire span is raised with jacks to release the cables and is lowered on the bridge seats. This method of erection is applicable to a half-through bridge with any number of spans. From November, 1914, to May, 1919, 152 bridges of this type were constructed in France.

The Ludwigshafen-Mannheim Bridge over the Rhine River, in Germany, was designed to be erected in two steps:[9] (1) Erect the bridge as a three-span, through-type structure with the trusses of uniform depth and continuous over the intermediate piers; and (2) construct a steel arch over the three spans, and reinforce the trusses, and suspend them to the arch. The result,

NOTE.—The paper by A. F. Reichmann, M. Am. Soc. C. E., was presented at the meeting of the Construction Division, New York, N. Y., January 16, 1930, and published in December, 1931, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1932, by Messrs. Howard P. Treadway, Benjamin W. Guppy, R. A. Van Ness, and J. G. Tripp.

[7] Sacramento, Calif.

[7a] Received by the Secretary August 10, 1932.

[8] *Le Genie Civil*, June 14, 1919.

[9] *Die Bautechnik*, September 16, 1930, p. 624.

after removing the intermediate piers, is a steel arch bridge with stiffening trusses, which forms an exceedingly graceful structure.

Mr. Reichmann has given a valuable summary of the construction plant and methods used in erecting steel bridges.

A. F. REICHMANN,[10] M. AM. SOC. C. E. (by letter).[10a]—The discussion has added some interesting cases of specific erection problems to a paper which was limited to a generalization of major considerations in the determination of the most feasible and economical equipment and methods of erection.

The writer wishes to express his appreciation to the discussers for their interest and commendation.

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[10] Asst. Chf. Engr., Am. Bridge Co., Chicago, Ill.

[10a] Received by the Secretary August 27, 1932.

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DISCUSSIONS

FULTON STREET, EAST RIVER TUNNELS, NEW YORK, N. Y.

Discussion

BY MILES I. KILLMER, M. AM. SOC. C. E.

MILES I. KILLMER,[13] M. AM. SOC. C. E. (by letter).[13a]—Mr. Feld comments on the omission of any description of the methods of running line. This phase of the work was done by the engineers of the Board of Transportation of the City of New York. It involves triangulation, base-line measurement, dropping line down the shafts, prolonging line through the lock bulkheads, and many other special methods. It is an extensive subject, and it could not be treated adequately, except in a separate paper. The work was done with great accuracy by the Engineers of the Board, and the Manhattan and Brooklyn survey lines checked at the junction of the shields within $\frac{1}{4}$ to $\frac{1}{2}$ in.

The discussion by Mr. Hatch requires some comment. The air pressure in a sand heading does not support the ground. The pressure used is such as to balance the head of ground-water at the bottom of the tunnel and is not influenced by the presence or absence of near-by building loads. The actual pressure in the land headings was generally not more than 18 lb. The 48-lb. pressure was required only in putting down the sumps below the tunnels at the lowest point under the river. The ground is supported by the cast-iron tunnel lining surrounded by its envelope of ejected pea gravel and grout, by the exterior of the skin on the shield, and, at the face, by breast-boards held in place by braces against the shield. When the shield is to be advanced, vertical timbers ("soldiers") are placed against the breast-boards. From these "soldiers," horizontal stretchers extend back through the pockets of the shield to cross-timbers ("walking sticks") wedged into the last cast-iron ring erected. The braces that had originally supported the breast against the shield are then knocked out, and the shield can be advanced without disturbing the support of the face. Under certain conditions, the horizontal

NOTE.—The paper by Miles I. Killmer, M. Am. Soc. C. E., was presented at the meeting of Construction Division, New York, N. Y., January 16, 1930, and published in December, 1931, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1932, by Messrs. H. J. King, S. M. Swaab, and Jacob Feld; and August, 1932, by Messrs. Ole Singstad, John F. O'Rourke, and H. H. Hatch.

[13] Mgr., Mason & Hanger Co., Inc., New York, N. Y.

[13a] Received by the Secretary August 23, 1932.

tables can be used to support the face. They are shoved out by means of their rams until they make contact with the "soldiers." When the shield is advanced the hydraulic pressure behind the rams is released slowly, and the tables come back as the shield goes ahead.

All the foregoing applies to a sand or gravel heading. If the face is of semi-plastic clay, a condition arises in which the pressure of air in the tunnel may be utilized to support the face, or at least to supplement other methods of supporting it.

The possibility of using sand with the pea gravel to secure an enveloping back-fill having a smaller percentage of voids, is mentioned by Mr. Hatch. This idea was successfully put in practice in the construction of the traffic tunnel from Boston to East Boston, Mass., recently completed. It was found that about one-third sand could be used with the pea gravel; a larger percentage caused stoppages in the tank or in the hose. In the Boston Tunnel, the surrounding ground was a stiff clay that did not fall into the void left by the advancing tail of the shield. It is the writer's opinion that where the surrounding ground is sand or gravel and tends to fall almost immediately into the annular space, pea gravel without sand will prove more effective as its granular nature makes it more fluid or mobile. The air pressure used in ejecting the gravel was the same as that used in grouting, that is about 80 lb.

Mr. Hatch also suggests the use of neat-cement grout to secure greater penetration. This was tried on a subsequent job with which the writer was connected. In driving two shields up Jay Street, Brooklyn, a very loose running sand and gravel was encountered. Some heavy buildings had to be passed. The quantity of 1 to 1 grout ejected was not fully satisfactory and neat-cement grout was resorted to for a distance of several hundred feet. The computed volume of the grout actually ejected, proved to be about equal to the computed volume of the 1 to 1 grout previously ejected, per foot of tunnel. In this case, therefore, the extra expense was not warranted. Of course, in grouting rock seams, neat-cement grout would probably be superior to the 1 to 1 mix.

Although the question of supporting buildings along a street seems to have been solved from an engineering point of view, there still remains the problem of defending suits brought by owners who, in many instances, will seize upon the occasion of the presence of a contractor to bring suit even when the physical damages are so slight as to require magnifying glasses to be seen.

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DISCUSSIONS

DETERMINATION OF PRINCIPAL STRESSES IN BUTTRESSES AND GRAVITY DAMS

Discussion

By Messrs. H. E. von Bergen, and Howard L. Cook

H. E. von Bergen,[25] Jun. Am. Soc. C. E. (by letter).[25a]—The author deserves much credit for his industry in deriving formulas for determining the principal stresses in gravity and buttressed dams. These formulas are rather tedious to develop as well as to apply and, therefore, it is interesting to note that, for a strictly triangular profile with water surface at the apex, the author also has shown (assuming a linear distribution of vertical normal stress) that the horizontal normal stress as well as the horizontal and vertical shear are distributed linearly. For this case, then, which occurs frequently, it is a simple matter, knowing the stresses at the up-stream and down-stream toes, to find the direction and magnitude of the principal stresses in the interior of a dam section or buttress by the application of Mohr's circle.

For the usual buttressed dam with a considerable inclination of the up-stream face, tension is found to exist at or near that face and is approximately parallel to it even if the vertical normal stress is of considerable magnitude in compression. This tension and the stress produced by the difference in shrinkage in the deck and in the supporting buttress, due to water-soaking, setting variation in temperatures, etc., are probably the two main causes for the typical "diagonal" cracks so frequently found in structures of this type.

It may be shown mathematically that, other things being equal, the tension due to the external loads on a buttressed dam—that is, the second principal stress—is a function of the slope of the up-stream face. Consider a buttressed dam and for simplicity assume that it has a weightless deck. The second principal stress at the up-stream face is:

$$f_2 = p_u (1 + n^2) - f_1 n^2 \quad \dots\dots\dots\dots\dots\dots (126)$$

Note—This paper by W. H. Holmes, Assoc. M. Am. Soc. C. E., was published in January, 1932, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1932 by Messrs. W. J. Stich, Hakan D. Birke, Dirk A. Dedel, Fred A. Noetzli, and Eugene Kalman; and August, 1932, by I. M. Nelidov, Assoc. M. Am. Soc. C. E.

[25] Div. of Water Resources, Dept. of Public Works, Sacramento, Calif.

[25a] Received by the Secretary August 29, 1932.

in which, p_u is the vertical normal stress at the up-stream face; f_1, the first principal stress at this point; and n, the slope of the up-stream face, or the cotangent of the angle the up-stream face makes with the horizontal.

By substituting in the equation:

$$p_u = \frac{\Sigma V}{A} - \frac{\Sigma M}{S} \qquad (127)$$

the physical dimensions of the buttress and the water loads, and by placing this value of p_u in Equation (126) and setting its first derivation with respect to n equal to zero, the following equation will result:

$$n^3 - A\,n^2 + B\,n - C = 0 \qquad (128)$$

in which, $A = \dfrac{6\,SL - 18\,y\,z\,t\,L\,K}{-\,4\,h\,S}$; $B = 1 + \dfrac{SL^2 - y\,t\,k\,L^2\,(1 + 3\,z)}{2\,h^2\,S}$; and $C = \dfrac{A}{3}$.

Furthermore, k = ratio of weights of a unit of masonry to a unit of water; h = height of buttress and depth of water; S = span of buttress; L = length of base of buttress; t = thickness of base of buttress; y = coefficient which, when multiplied by $h\,L\,t$, gives the volume of the buttress; and z = coefficient, which when multiplied by h, gives the vertical distance from the center of gravity of the buttress to its base.

For a numerical example consider a buttress in which the height and length of base equal 100 ft., the thickness is uniformly 3 ft., and for which $y = \frac{1}{2}$, $z = \frac{1}{3}$, and the span of the buttresses is 30 ft. For the water surface at the crest the solution of Equation (128) for this example will give a value of n equal to 0.5 which is the slope, as far as the water load and weight of buttress are concerned, that will produce a minimum tension parallel to the up-stream face. For the case in which the buttress thickness decreases to zero at the apex n is equal to 0.48. (Fredrik Vogt, Assoc. M. Am. Soc. C. E., states[26] that the best slope is probably 65 to 70°; that is, n = 0.37 to 0.47.) These slopes will be altered slightly upon inclusion of the deck weight or alteration of physical dimensions. In the writer's opinion the foregoing consideration points more directly toward the "design of buttresses eliminating tension," than the argument in the author's paper.

To determine tensile stress in the concrete caused by the water-soaking of the deck and the shrinkage of the adjacent buttress due to dissipation of setting heat is a matter of conjecture. (This consideration applies only to the case in which the deck is fixed to the up-stream face of the buttress.) As an example, assume a 40° drop in temperature in the buttress after setting, with the accompanying shrinkage and a swelling of the deck due to water-soaking of 0.005%;[27] and assuming a modulus of elasticity at 2 000 000 and one-half the deformation as compression in the arch, the tensile stress in the buttress parallel to the up-stream face is 270 lb. per sq. in. In summer, when the temperature of the buttress is considerably higher, the stress will be less, and *vice versa.*

[26] In "Economical Design of Buttresses for High Dams and Cellular Gravity Dams."

[27] Rept., U. S. Bureau of Reclamation, Vol. 1, Section 1, Compilation of Data on Concrete Problems.

That temperature greatly affects the deformation of a buttress is shown by data obtained on variations in width of the diagonal cracks in the buttresses of Lake Hodges Dam, near San Diego, Calif. In one instance, neglecting the gradual increase in the width of the crack, it was found that a drop in temperature of 20° Fahr. was accompanied by an increase in width of the crack, in a direction normal to it, of 0.058 in., while the reservoir stage was fairly constant. In other instances the data showed no obvious variation in width of crack with the fluctuation of the water surface. There did seem, however, to be some slight deformation parallel to the crack with this fluctuation. This is probably due to the fact that a drop in temperature produces contraction along the three major axes, whereas stress induced by loading does not necessarily cause such deformation.

From the foregoing discussion, it seems obvious that unsightly cracks can not be eliminated by simply eliminating tension due to external loading. Either there must be sufficient reinforcing steel to force the buttress to act as a structural monolith, or contraction joints must be provided similar to those in Figs. 24 and 25.[27a]

The writer believes another matter worthy of discussion is the quantity and manner of placing steel reinforcement in a buttress. Some buttressed dams have been built with vertical and horizontal steel, some with inclined and horizontal steel, and some with no steel at all. After a few years of service, the buttress usually shows into which class it falls.

Textbooks on the subject of reinforced concrete usually indicate that, in a homogeneous beam, ideal reinforcement follows the trajectories of the second principal stress. This same reasoning might well hold good in a buttress.

The percentage of steel necessary in this case, if the direction of the steel and tension coincide, may be computed simply by dividing the computed tension by the allowable stress in steel, or $p = \frac{f_2}{f_s}$; or, if the steel lies in two different directions (see Fig. 27), the stress in the horizontal steel is:

$$f_h = \frac{f_2}{p_h \cos\alpha\,[\cos\alpha + \sin\alpha\cot(\beta - \alpha)]} \quad \text{.............} (129)$$

and the stress in the inclined steel is:

$$f_i = \frac{f_2}{p_i \cos(\beta - \alpha)\,[\cos(\beta - \alpha) + \sin(\beta - \alpha)\cot\alpha]} \quad \text{........} (130)$$

It will be seen that if $\beta = 90°$ and $\alpha = 45°$, the steel is just one-half as effective as when it coincides with the direction of the tension and $\beta = \alpha = 0$. These equations are based on the assumption that the concrete has cracked normal to f_2, and that the steel carries the tension. The percentages of the horizontal and inclined steel are represented by p_h and p_i, respectively.

An analogy may be made between web reinforcement in a concrete beam and reinforcement in a buttress; assume that the vertical stirrups carry the

[27a] *Proceedings*, Am. Soc. C. E., May, 1932, p. 874.

total shear.[28] Actually, the vertical stirrups are assumed to carry the vertical component of the diagonal tension which, numerically, is of the same magni-

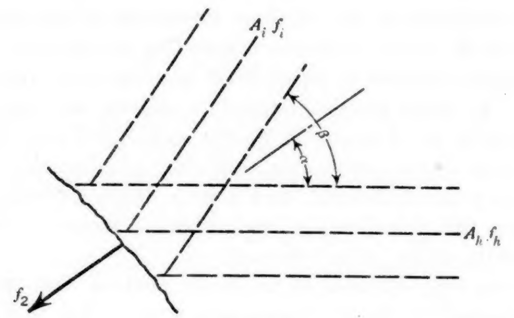

FIG. 27.

tude at the neutral plane. In Equation (127) the $\frac{2}{3}$ is omitted since steel carries all the tension; thus, $A_s = \frac{V_s}{f_s j d}$; $V = v'\, b\, j\, d$; and $A_s = \frac{v'\, b j d s}{f_s j d}$. In terms of p, the ratio of steel area to concrete area is $A_s = p\, b\, s$; and substituting f_2, the "diagonal tension," for v', its numerical equivalent,

$$f_s = \frac{f_2}{p} \qquad \text{(131)}$$

For inclined stirrups[29], or bars bent up at 45° (omitting the $\frac{2}{3}$), $A_s = \frac{0.7\,(V_s)}{f_s j\, d}$. In terms of p, again, $A_s = p\, b\, s \sin 45°$, and,

$$f_s = \frac{f_2}{p} \qquad \text{(132)}$$

as in Equation (131).

It will be found that by substituting in Equation (130), $\beta = 90°$ and $\alpha = 45°$ (which corresponds to vertical stirrups), that Equation (131) will result; and by substituting in Equation (130) $\beta = 45°$ and $\alpha = 45°$ (which corresponds to bars bent up at 45°) that Equation (132) is the result. It will also be found in the latter case that the inclined steel carries the total tension, leaving no horizontal component to be carried by the horizontal steel and, therefore, Equations (129) and (130) are consistent for web reinforcement. In the former case, involving vertical steel, it will be found that the horizontal steel carries the horizontal component of the diagonal tension produced by shear, but that it is considered as bond stress in reinforced concrete beams. Thus, the analogy is complete.

Temperature steel either may be considered separate or, if convenient, may be combined with other steel to resist stresses due to the combination of temperature and external loads. The steel necessary to prevent large cracks that are likely to occur, due to shrinkage of a member restrained at both

[28] "Reinforced Concrete Handbook," by Hool and Johnson, p. 285.

[29] *Loc. cit.*, Equation (4), p. 286.

ends, ranges up to about 0.4%, depending on the maximum tensile strength of the concrete.

As shear always resolves itself into tension and compression it seems more sensible to reinforce against the tension directly instead of its components because concrete is by far the weakest in tension, whereas its pure shearing strength is from 60 to 80% of the compressive strength. In other words, with the diagonal tension adequately provided for, shear steel or shear reinforcement is unnecessary and irrelevant.

HOWARD L. COOK,[30] JUN. AM. SOC. C. E. (by letter).[30a]—The formula, $\frac{P}{a} \pm \frac{Mc}{I}$, for the normal stress over a section of a bent and compressed beam (Equation (1)) is applicable when: (1) The dimensions of the sections are small in comparison with the length of the beam; (2) these dimensions are small compared to the radius of curvature of the central line; (3) the cross-sections considered are planes perpendicular to the line of their centroids both before and after bending; and (4) Hooke's law applies to all the material affected, and this material is homogeneous and isotropic.

A masonry dam is a thick, short beam, formed so that Conditions (1), (2), and (3), are not met. Condition (4) may be allowed for purposes of discussing the action of such a beam. Obviously, the formula given does not apply to a horizontal, or any other, section of such a structure.

Equation (1) gives a linear distribution of normal stress over a section of the dam. If it does not apply, the so-called law of the trapezoid does not hold for a dam. Mr. Holmes recognizes that this is so in his statement (under "Distribution of Stresses") that, "it is well known that the vertical normal stress distribution of gravity dams is not linear, although for triangular dams of thicknesses of 50 to 60 ft., the error is small." Evidently, he thought the law held closely enough to enable a complete internal stress analysis to be based upon it. The question is: Does it hold with sufficient exactness to support an analysis of any scientific utility whatsoever?

There is only one way of getting at the answer to this question; that is, by experiments on model dams. The late William Cain, M. Am. Soc. C. E., in the paper upon which Mr. Holmes bases his work,[31] refers to a number of these experiments, all of which happen to have been made in England. Professor Cain's conclusion was that these investigations substantiated the view that the stress distribution is closely linear for horizontal sections not too close to the foundation.

In a paper by Mr. B. F. Jakobsen,[32] the linearity of the stress distribution at horizontal sections is not admitted. Indeed the attempt is made to find the true distribution. Mr. Jakobsen also refers to the experiments cited by

[30] Asst. Engr. with Robert E. Horton, M. Am. Soc. C. E., Cons. Engr., Voorheesville, N. Y.

[30a] Received by the Secretary August 29, 1932.

[31] "Stresses in Masonry Dams," *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), p. 208.

[32] "Stresses in Gravity Dams by Principle of Least Work," *Proceedings*, Am. Soc. C. E. September, 1930, Papers and Discussions, p. 1613.

Professor Cain, calling them the "English" experiments. It is from these experiments that Mr. Jakobsen arrives at the conclusion that substantial departures from linearity of stress distribution actually occurs.

If the hypothesis of linearity of stress distribution is substantially in error for gravity dams, it is misleading to base a complete internal stress analysis upon it. If Mr. Holmes errs in doing this, it must be said that he only follows a popular fashion among engineers.

A careful investigation of the internal strains and stresses in models of dams was made in 1907 by Messrs. Karl Pearson and A. F. C. Pollard.[33] It demonstrated more clearly than any other, the alarming discrepancy between actual normal stress distributions and the hypothetical linear distribution. A peculiarity of this investigation is the lack of notice it receives from engineers. In 1904, Professor Pearson, with Mr. L. W. Atcherley[34] made some elementary experiments on models of dams. These experiments were, and still are, widely discussed in engineering texts and papers dealing with the design of dams. The 1907 series of experiments under the supervision of Professor Pearson, while of much greater value, are seldom mentioned.

At the suggestion of the late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., Messrs. Pearson and Pollard adopted a jelly as the material for their models. It consisted of gelatine, glycerine, and coloring matter, and was cast into shape in moulds. The models adopted were about 18 in. through the base and 14 in. high. The length of a section was about 4 in. The models were of shapes similar to the Vyrnwy Dam, in Wales, and the Assuan Dam, in Egypt.

Two kinds of experimentation were pursued. A first series of models were ruled with a grid of lines 2 cm. apart. Forces were applied to the models and full-sized photographs obtained under various conditions of loading. Angle changes in the grid system were then measured on the photographs by a specially designed instrument called a direct microgoniometer.

The final experiment of the investigation was made with extreme care. A model of the Assuan Dam was ruled while just floating in a bath of glycerine, thus eradicating stresses due to the weight of the dam. This model was tested directly at various loads by an optical microgoniometer developed for the work.

Normal water loads on the models were supplied by means of an elastic bag of liquid resting against the up-stream face. Exaggerated loadings were made by means of a board pressing on the up-stream face of the jelly model and loaded normally at a depth of two-thirds the total.

If the hypothesis of linear distribution of stress were true, the shearing stresses would be distributed parabolically. The experimenters simply had to compare the actual shear distribution of their models with a parabolic distribution, to determine the applicability of the linear hypothesis.

[33] "An Experimental Study of the Stresses in Masonry Dams," Drapers Company *Research Memoirs*, Tech. Series V, Cambridge Univ. Press.

[34] "On Some Disregarded Points in the Stability of Masonry Dams," by L. W. Atcherley, Assisted by Karl Pearson, Drapers Company *Research Memoirs*, Tech. Series II.

As the shearing stress is proportional to the slide or change of angle in the grid system at any point, measurement of this angle change was all that was required. Details of methods and results may be had by referring to the paper[33] of Messrs. Pearson and Pollard.

As a result of their work the experimenters arrived at conclusions which, in substance, follow:

(1) There is no approach to a parabolic distribution of shear, at least up two-thirds, and probably up the entire height, of masonry dams. Linearity of distribution of vertical stresses on horizontal planes is a wholly unjustifiable hypothesis.

(2) The present theory of the stability of masonry dams is erroneous. The line of resistance being within the middle third does not prevent the material from being in tension. The customary theoretical analysis adds an appearance of safety not warranted by experiment, for the stresses obtained thereby are probably not within 100% of those in the actual dam.

(3) Tension exists in masonry dams of the types commonly constructed, but is not necessarily a source of danger. It seems desirable to admit that masonry can and does undergo tension and to set some limit not to be exceeded by these stresses in any dam.

(4) It is possible by experimenting upon a model dam to determine stresses within 10% of those in a theoretical dam and within 30 to 40% of those in the actual structure. Model investigations should be made the basis of the design of high masonry dams.

Having determined the approximate shape of the shear curves a semi-empirical determination of the stress in the body of masonry dams was attempted. The undulating character of the curve of shear stress required that, at the very least, a quartic distribution of shear be assumed. This would make the normal stress-distribution curve a cubical one. After considerable labor on the solution of the complicated equations involved, the problem still remained without a satisfactory answer. It was found, however, that the quartic shear curve would: (1) Satisfy the internal equations of elasticity; (2) satisfy the stress conditions at the up-stream face; and (3) bring into equilibrium the resultant force and stresses on horizontal sections at top, mid-height, and base of dam. The method does not account for the true distribution of stresses over the down-stream face, however.

Compare the merits of the foregoing solution with that of the paper under discussion. The solution given by Mr. Holmes may be made under certain conditions: (*a*) To satisfy the surface stress conditions at the up-stream face; and (*b*) to satisfy the equilibrium of stress and resultant force on the base section only. The stress distribution on any horizontal section by this solution will be totally unlike that found by experiment. In comparison with the old middle-third theory, however, the analysis outlined by Mr. Holmes is an advance. The middle-third hypothesis fails to satisfy stress conditions at the up-stream or the down-stream face, or at the top of the dam. It also gives linear distributions of normal stress on horizontal sections.

Even if Mr. Holmes' analysis is preferable to the use of the old rule of the middle third, it seems far-fetched to extend it to the point of predicting completely the stresses throughout a dam. Such an elementary treatment should never pretend to the dignity of a complete analysis. In view of the departures from a linear distribution of stress shown by experiment, it is doubtful whether such an analysis is even useful. It would seem dangerous to locate joints, as suggested, along the stress paths calculated by the methods given.

Messrs. Pearson and Pollard, in 1907, recommended the experimental investigation of a model of any proposed dam. To-day, the Engineering Profession is just entering the era of model investigation.

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DISCUSSIONS

STRESSES IN INCLINED ARCHES OF MULTIPLE-ARCH DAMS

Discussion

BY MESSRS. VICTOR H. COCHRANE, PAUL BAUMAN, LARS R. JORGENSEN, FRED A. NOETZLI, AND W. J. STICH

VICTOR H. COCHRANE,[9] M. AM. SOC. C. E. (by letter).[9a]—The authors are to be commended for their clear and complete solution of the problem of inclined arches for dams. The writer appreciates the great amount of labor involved in the preparation of this paper, as he has had occasion to develop similar data for use in connection with the design of a large multiple-arch dam having a central angle of 160° and a buttress spacing of 70 ft. In this work he had the assistance of Donald Witten, M. Am. Soc. C. E.

The writer's formulas were derived for precisely the same conditions as those used by the authors, except that nothing was done in regard to the hinged arch, which seems to have little or no application in multiple-arch dams. In the development of formulas for the inclined arch the chief difficulty lies in the derivation of expressions for the horizontal thrust at the crown. All other formulas required are comparatively simple. The shears are always small, and, ordinarily, they do not need to be computed. The writer's formulas for the horizontal thrust were derived by a method that differs entirely from that used by the authors, and the two sets of formulas appear at first glance to be very different. However, a comparison of charts by both methods seems to show that they give the same results. For the uniform water load it is found that the expression derived by the late William Cain, M. Am. Soc. C. E., can be reduced to that derived by the writer.

At first, it was intended to prepare a set of curves similar to those presented by the authors, but this plan was not fully carried out. It was found that the data are better suited to tables than to diagrams, for the reason

NOTE.—The paper by George E. Goodall and Ivan M. Nelidov, Associate Members, Am. Soc. C. E., was published in March, 1932, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[9] Cons. Engr., Tulsa, Okla.

[9a] Received by the Secretary March 31, 1932.

that nearly all the terms in the various formulas are independent of the mean radius or the thickness of the rib, being functions of the central angle alone. If the central angle is kept constant, or nearly so, the values of these terms are good for any section of the arch. Those terms involving the radius or thickness are few, as will be shown. Tables can readily cover a wider range than curves. The former are the more accurate. They are more easily prepared and are more convenient to use. A compact table such as Table 2, when used in connection with the formulas of Table 3, is substantially the equivalent of the authors' numerous curves.

If no curves are at hand and the central angle has an odd value not included in the table, it is quite easy to compute the required constants with the aid of a calculating machine, provided the formulas are simplified and the work is systematically arranged. In the example given herein the cal-

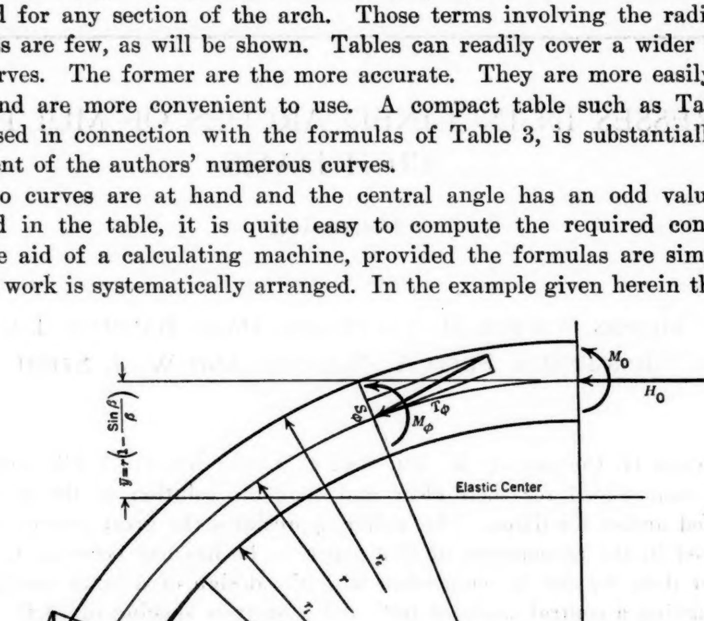

FIG. 12.

culations are set forth in full, and it will be seen that the work involved is far from formidable. The computation of the required values takes but little more time than is needed for reading the curves.

Four load conditions and corresponding constants are expressed as follows:

Load 1.—Arch Weight:

$$K_1 = w_0\, t\, r \cos \alpha \quad \text{(75a)}$$

Load 2.—Uniform Water Load:

$$K_2 = w\, h\, r_e \quad \text{(75b)}$$

Load 3.—Variable Water Load:

$$K_3 = w\, r_e^2 \cos \alpha \qquad (75c)$$

Load 4.—Temperature Change:

$$K_4 = \varepsilon\, \Delta_t\, E \qquad (75d)$$

In Equation (75*d*), Δ_t is positive for a rise in temperature. Equations (75) are the same as the load constants given by the authors, except that r_e (Fig. 12) is used in Equation (75*b*) as well as in Equation (75*c*).

TABLE 2.—VALUES OF FUNCTIONS OF β

β	A	B	C	D	F	J	Q	S
60	3.24907	0.04782	0.89922	0.18298	0.04025	0.00378	0.15399	0.00951
65	3.68164	0.06941	1.04551	0.26419	0.05650	0.00645	0.17516	0.01298
70	4.13609	0.09760	1.18186	0.36929	0.07650	0.01055	0.19628	0.01729
75	4.60891	0.13346	1.30061	0.50180	0.10027	0.01660	0.21701	0.02254
80	5.09600	0.17807	1.39408	0.66498	0.12758	0.02524	0.23699	0.02885

β	$1 - \frac{\sin\beta}{\beta}$	$\beta \sin\beta + \cos\beta - 1$	$\sin^2 \frac{\beta}{2}$	$\sin\frac{\beta}{2} \cos\frac{\beta}{2}$	$1 - \frac{\beta \sin\beta}{2} - \cos\beta$	$\sin\beta - \beta\cos\beta$	$\frac{\sin\beta}{\beta} - \cos\beta$
60	0.17301	0.40690	0.25000	0.43301	0.04655	0.34242	0.32699
65	0.20111	0.45079	0.28869	0.45315	0.06330	0.42687	0.37627
70	0.23085	0.49007	0.32899	0.46985	0.08396	0.52183	0.42713
75	0.26209	0.52321	0.37059	0.48296	0.10899	0.62714	0.47909
80	0.29468	0.54870	0.41318	0.49240	0.13883	0.74235	0.53167

The necessary formulas for thrusts, moments, and shears are arranged for convenience in Table 2, for the four load conditions. The constants for substitution in Table 2 are:

$$A = \beta\ (m + 1) - \frac{m-1}{2} \sin 2\beta \qquad (76a)$$

$$B = \beta + \frac{\sin 2\beta}{2} - \frac{2 \sin^2 \beta}{\beta} \qquad (76b)$$

$$C = \frac{m-1}{4} \sin 2\beta - 2\beta \cos 2\beta \qquad (76c)$$

$$D = \frac{m+2}{2}\beta - \frac{3\,\mathrm{m} - 5}{8} \sin 2\beta + \frac{m-1}{4} \beta \cos 2\beta - 2 \sin\beta \qquad (76d)$$

$$F = \frac{4 \sin^2\beta}{\beta} - \beta \sin^2\beta - \frac{7}{4} \sin 2\beta - \frac{\beta}{2} \qquad (76e)$$

$$J = \frac{3}{4}\beta + \frac{9}{8} \sin 2\beta + \frac{\beta \sin^2\beta}{2} - \frac{3 \sin^2\beta}{\beta} \qquad (76f)$$

$$Q = \frac{2 \sin\beta}{\beta} - \cos\beta - 1 \qquad (76g)$$

and,

$$S = 1 + \frac{\cos\beta}{2} - \frac{3}{2}\,\frac{\sin\beta}{\beta} \qquad (76h)$$

The factor, m, depends upon the ratio, $\frac{E}{G}$, and the shear distribution; the value, 2.88, may be used. If $m = 2.88$, the terms, A, C, and D, in Equations (76) reduce to the following:

$$A = 3.88 - 0.94 \sin 2\beta \quad \text{(77a)}$$

$$C = 0.47 (\sin 2\beta - 2\beta \cos 2\beta \quad \text{(77b)}$$

and,

$$D = 2.44\beta - 0.455 \sin 2\beta + 0.47\beta \cos 2\beta - 2 \sin \beta \quad \text{(77c)}$$

In practice, the writer arranges Fig. 12, Equations (75) and (76), and Table 2 on a single sheet for ready reference. Having computed the constants, A, B, C, D, F, J, Q, and S for the central angle chosen, the formulas for H_0 and M_0 become very simple. It will be noted that all the expressions for

TABLE 3.—FORMULAS FOR THRUSTS, MOMENTS, AND SHEARS

Load	$H_0 =$	$M_0 =$	T_ϕ and S_ϕ (for springing section use β for ϕ)	$M_\phi =$
1	$K_1 \frac{C + uF}{A + uB}$	$K_1 r\, Q - H_0 \bar{y}$	$T_\phi = K_1 \phi \sin\phi + H_0 \cos\phi$ $S_\phi = K_1 \phi \cos - H_0 \sin\phi$	$M_0 + H_0 r\, (1 - \cos\phi)$ $- K_1 r\, (\phi \sin\phi + \cos\phi - 1)$
2	$K_2 \frac{A - 2\sin\beta + uB}{A + uB}$	$(K_2 - H_0)\, \bar{y}$	$T_\phi = 2 K_2 \sin^2 \frac{\phi}{2} + H_0 \cos\phi$ $S_\phi = 2 K_2 \sin \frac{\phi}{2} \cos \frac{\phi}{2} - H_0 \sin\phi$	$M_0 + H_0 r\, (1 - \cos\phi)$ $- 2 K_2 r \sin^2 \frac{\phi}{2}$
3	$K_3 \frac{D + uJ}{A + uB}$	$K_3 r S - H_0 \bar{y}$	$T_\phi = K_3 \left(1 - \frac{\phi \sin\phi}{2} - \cos\phi\right) + H_0 \cos\phi$ $S_\phi = \frac{K_3}{2} (\sin\phi - \phi\cos\phi) - H_0 \sin\phi$	$M_0 + H_0 r\, (1 - \cos\phi)$ $- K_3 r \left(1 - \frac{\phi \sin\phi}{2} - \cos\phi\right)$
4	$K_4 t \frac{2 \sin\beta}{A + uA}$	$- H_0 \bar{y}$	$T_\phi = H_0 \cos\phi$ $S_\phi = - H_0 \sin\phi$	$H_0 r \left(\frac{\sin\beta}{\beta} - \cos\phi\right)$

H_0 in Table 3 have the same denominator, $A - uB$, in which, $u = 12\left(\frac{r}{t}\right)^2$. For example, if $\beta = 75°$; $\frac{t}{r} = 0.30$; and $u = 133.33$; from Table 2:

Dead load,

$$H_0 = \frac{14.67}{22.41} K_1 = 0.655 K_1 \quad \text{(78a)}$$

Uniform water load,

$$H_0 = \frac{22.41 - 1.93}{22.41} K_2 = 0.914 K_2 \quad \text{(78b)}$$

Variable water load,

$$H_0 = \frac{2.715}{22.41} K_3 = 0.1212 K_3 \quad \text{(78c)}$$

Fall in temperature,

$$H_0 = - K_4 t \frac{1.93}{22.41} = - 0.086 K_4 t \quad \text{(78d)}$$

These values all agree with those shown in the authors' diagrams. Some of the coefficients for T_β, M_β, and S_β, are given in Table 2, which illustrates a part of a list of values of functions for half-central angles at 5° intervals. The table used in practice extends from $\beta = 35°$ to $\beta = 90°$, and other values may be found by interpolation if for any reason the central angle chosen is not included. It will be noted in Table 3 that, with the exception of the "load constants," the only terms that vary with the mean radius or thickness of the rib are $\bar{y}$ and u. In the expression for horizontal thrust due to the uniform water load, the terms, A, in the denominator, and, $A - 2 \sin \beta$, in the numerator, represent the effect of shear and arch-shortening. If these terms are dropped, $H_0 = K_2$, which is the so-called cylinder formula. If shear and direct stress are taken into account, the coefficient of K_2 must be less than unity. The expression, $H_0 - K_2$, is the force applied at the elastic center in combination with the cylinder thrust, K_2. In the expressions for H_0 for variable water load and dead load, the first terms of the numerator and denominator represent the effect of shear and direct stress, and the last terms show the influence of bending moments. When numerical substitutions are made, the relative influence of shear and direct stress, as compared with bending moments, is readily seen. The effect of the curvature of the arch was investigated and found to be negligible for values of $\frac{t}{r}$ up to to 0.30, or greater.

In order to illustrate the use of these formulas, calculations are given for the authors' example, in which $\beta = 71° \; 45' = 1.25227$ radians. The trigonometric values required are, as follows: $\sin \beta = 0.94970$; $\cos \beta = 0.31316$; $\sin 2\beta = 0.59482$; $\cos 2\beta = -0.80386$; $\sin \frac{\beta}{2} = 0.58602$; $\cos \frac{\beta}{2} = 0.81030$; $\sin^2 \beta = 0.90193$; $\sin^2 \frac{\beta}{2} = 0.34342$; $\beta \sin \beta = 1.18928$; $\beta \cos \beta = 0.39216$; $\beta \sin^2 \beta = 1.12946$; $\frac{\sin \beta}{\beta} = 0.75838$; $\frac{\sin^2 \beta}{\beta} = 0.72073$; $\beta \cos 2\beta = -1.00665$; $\sin \beta \cos \beta = 0.29741$; $\sin \frac{\beta}{2} \cos \frac{\beta}{2} = 0.47485$; $\beta \sin \beta + \cos \beta - 1 = 0.50244$; $1 - \frac{\beta \sin \beta}{2} - \cos \beta = 0.09220$; $\sin \beta - \beta \cos \beta = 0.55754$; and, $\frac{\sin \beta}{\beta} - \cos \beta = 0.44522$.

The functions of β are (from Equations (76)):

$$A = 3.88 \times 1.2523 - 0.94 \times 0.59482 = 4.300;$$
$$B = 1.2523 + 0.29741 - 1.44046 = 0.1093;$$
$$C = 0.47\,(0.59482 + 2.01330) = 1.2258;$$
$$D = 2.44 \times 1.2523 - 0.455 \times 0.59482 - 0.47 \times 1.00665 - 1.89940 = 0.41244;$$
$$F = 2.88092 - 1.12946 - 1.75 \times 0.59482 - 0.62613 = 0.08439;$$
$$J = 0.93920 + 1.125 \times 0.59482 + 0.56473 - 2.16069 = 0.01241;$$
$$Q = 1.51676 - 1.31316 = 0.20360; \text{ and,}$$
$$S = 1.15658 - 1.13757 = 0.01901.$$

For $r = 25.79$; $t = 2.58$; h (depth over crown) $= 55.32$; $u = 12 \times 10^2 = 1\,200$; and, $\bar{y} = 0.24162\ r = 6.231$; $K_1 = 150 \times 2.58 \times 25.79 \times 0.6820 = 6\,807$; $K_2 = 62.5 \times 55.32 \times 27.08 = 93\,630$; $K_3 = 42.6 \times 27.08^2 = 31\,240$; and, $K_4 = -\ 576\,000\,000 \times 20 \times 0.0000055 = -\ 63\,360$. Then: $K_1 r = 175\,550$; $K_2 r = 2\,414\,700$; and, $K_3 r = 805\,700$.

The terms required for computing the four values of H_0 are: $A - uB = 4.30 - 1\,200 \times 0.1093 = 135.46$; $C - uF = 1.226 - 1\,200 \times 0.08439 = 102.50$; and, $D - uJ = 0.41 - 1\,200 \times 0.01241 = 15.30$.

The values of thrusts, moments, and shears (with the authors' corresponding values in parentheses) are:

Load 1.—Weight of Arch:

$H_0 = 0.757 \times 6\,807 = 5\,150\ (5\,040)$; and $H_0 r = 132\,800$

$M_0 = 175\,550 \times 0.2036 - 5\,150 \times 6.231 = 3\,650\ (-\ 3\,560)$

$T_\beta = 6\,807 \times 1.1893 - 5\,150 \times 0.3132 = 9\,710\ (9\,475)$

$S_\beta = 6\,807 \times 0.3922 - 5\,150 \times 0.9497 = -\ 2\,220\ (-\ 2\,183)$

$M_\beta = 3\,650 + 132\,800 \times 0.6868 - 175\,550 \times 0.5024 = 6\,660\ (-\ 6\,605)$

Load 2.—Uniform Water Load:

$H_0 = 0.986 \times 93\,630 = 92\,320\ (92\,300)$; and $H_0 r = 2\,380\,900$

$M_0 = (93\,630 - 92\,320)\ 6.231 = 8\,160\ (-\ 8\,185)$

$T_\beta = 93\,630 \times 0.6868 - 92\,320 \times 0.3132 = 93\,220\ (93\,200)$

$S_\beta = 93\,630 \times 0.9496 - 92\,320 \times 0.9497 = 1\,240\ (1\,270)$

$M_\beta = 8\,160 + 2\,380\,900 \times 0.6868 - 2\,414\,700 \times 0.6868$
$= -\ 15\,050\ (-\ 14\,940)$

Load 3.—Variable Water Load:

$H_0 = 0.1129 \times 31\,240 = 3\,430\ (3\,535)$; and $H_0 r = 91\,040$

$M_0 = 805\,700 \times 0.01901 - 3\,530 \times 6.231 = (-\ 6\,715)$

$T_\beta = 31\,240 \times 0.0922 - 3\,530 \times 0.3132 = 3\,990\ (3\,990)$

$S_\beta = 15\,620 \times 0.5575 - 3\,530 \times 0.9497 = 5\,360\ (5\,355)$

$M_\beta = -\ 6\,680 + 91\,040 \times 0.6868 - 805\,700 \times 0.0922$
$= 18\,440\ (-\ 18\,470)$

Load 4.—Drop in Temperature:

$K_4 t = -\ 63\,360 \times 2.58 = -163\,470$

$H_0 = 0.01401 \times -\ 163\,470 = -\ 2\,290\ (-\ 2\,420)$

$M_0 = -\ 2\,290 \times 6.231 = -\ 14\,270\ (-\ 15\,120)$

$T_\beta = -\ 2\,290 \times 0.3132 = -\ 717\ (-\ 757)$

$S_\beta = +\ 2\,290 \times 0.9497 = +\ 2\,175\ (+\ 2\,300)$

$M_\beta = -\ 59\,060 \times 0.4452 = -\ 26\,290\ (-\ 27\,860)$

The authors call attention to the large bending moments for sections near the water surface. Unfavorable conditions may occur throughout the range of the draw-down, although elements some distance below the top of the dam derive support from adjacent elements above the water surface. At low water the arch in the vicinity of the water surface may actually be more severely stressed than it would be with the reservoir full.

Thickening of Arch at Haunches.—In the authors' example the temperature drop is 20 degrees. No mention is made of shrinkage, which acts like a fall in temperature. While the effect of shrinkage may be reduced by plastic deformation, it may be nevertheless much greater than that of the temperature change. Other factors tending to set up bending stresses in the arch are the water-soaking of the up-stream face and a difference in temperature between the two faces. All these conditions may be provided for by making a liberal allowance for temperature drop. In some recent bridge designs the equivalent temperature drop has been as much as 120 degrees. In order to determine the proper amount and location of the reinforcement, it is necessary to make an adequate allowance for temperature changes and shrinkage, with perhaps higher working stresses than are customary.

Owing to the large bending moments set up by changes in the length of the arch, the stresses will always be greatest at the crown and springing sections, and least at the plane of the elastic center, which is at the depth, $\overline{y}$, below the crown. The stress at the abutment section is much greater than that at the crown. For instance, consider the authors' example, and the same case with the temperature drop increased to 80° to include the effect

TABLE 4.—COMPARISON OF STRESSES AT CROWN AND ABUTMENT SECTIONS

Load	CROWN SECTION				SPRINGING SECTION			
	Thrust, in pounds	Moment, in foot-pounds	Maximum	Minimum	Thrust, in pounds	Moment, in foot-pounds	Maximum	Minimum
1...............	+ 5 150	+ 3 650	......	......	+ 9 710	+ 6 660	......	....
2...............	+92 320	+ 8 160	......	......	+93 220	—15 050	......	....
3...............	+ 3 530	— 6 680	......	......	+ 3 990	—18 440	......	....
4...............	— 2 290	+14 270	......	......	— 720	—26 290	......	....
Total, 20°....	+98 710	+19 400	— 387*	— 144*	+106 200	—53 120	— 618*	+ 47†
Total, 80°....	+91 840	+62 210	— 635*	+ 143†	+104 040	—131 990	—1 106*	+ 546†

* Minus = compression. † Plus = tension.

of shrinkage. For the purpose of making a rough comparison of the stresses at the crown and abutment sections, it is assumed in Table 4 that the arch is capable of resisting tensile stresses. For reinforced sections, making the usual assumption that the concrete carries no tension, the inequality of stresses will be even greater. For the 80° drop the arch would have to be reinforced, and the maximum springing stress would be about double the maximum crown stress.

These considerations lead inevitably to the conclusion that a fixed arch barrel of uniform thickness from crown to springing is inefficient, and should not be used. For a given working stress the crown section may be made thinner than the abutment section. In other words, the arch should be thicker at the abutment than at the center. The thickening should begin about as far below the elastic center as the crown is above it. The desired result can perhaps best be obtained by making the intrados three-centered, as shown in Fig. 12. The point of compound curve is at an angular distance, $\frac{\beta}{3}$, from the

springing. The radius of the end section is chosen so as to make the increase in thickness, $c\, r_i$, any desired amount. In order to simplify the form work, it is desirable to keep the intrados constant throughout the arch barrel. If this is done, $c\, r_i$ must also be constant. This gives a greater relative thickening for the thin arches near the top, which, fortunately, is what is needed. It will be noted that the arch axis shifts at the abutment a distance equal to one-half the thickening. The bending moment with reference to the new axis is changed by the amount, $\frac{c r_i}{2} T_\beta$.

In the case previously considered, assume that the springing section is thickened so that $c = 0.03$, and that the temperature drop is 80 degrees. Then, $c\, r_i = 0.03 \times 24.5 = 0.73$ ft.; and, $t = 2.58 + 0.73 = 3.31$ ft. Inasmuch as the arch is stiffer, the moments will be increased, but part of the increase will be offset by the shift of the axis. If the thrusts and moments for this case are assumed to be the same as for the unthickened arch, the approximate stresses for the thickened arch are found to be as follows: The direct stress is $+\frac{104\,040}{3.31} = +\,31\,430$; the bending stress is, $\pm\ 131\,990 \times \frac{6}{3.31^2} \pm\ 72\,280$; the combined stress, is pounds per square inch, is $-\ 720$ or $+\ 284$; and, as computed for the unthickened arch, it is $-\ 1\,106$ or $+\ 546$. The comparatively small thickening reduces springing stresses materially, and the reinforcement will no doubt be lessened.

The writer has investigated the effect of the proposed thickening on the thrusts and moments as computed for arches of uniform thickness. The method was to calculate the changes in crown deflections due to the increase in thickness, for various values of $\frac{t}{r}$, β, and c, using summations at 5° intervals. These values were plotted into curves for reference in design.

If the abutment section is thickened so as to make the crown and springing stresses approximately equal, the effect of the change on the stresses is not great, and may well be neglected. For example, if the arch previously considered is thickened in this manner, the maximum crown and springing stresses are, respectively, about 5% and 10% greater than those previously computed.

PAUL BAUMAN,[10] M. AM. SOC. C. E (by letter).[10a]—In the "Introduction," the authors state that the analytical method developed by Dr. N. Kelen[11] "provides curves from which the crown thrust may be obtained directly, but does not include curves or coefficients from which the abutment thrust, abutment shear, and bending moments may be directly obtained." They state also that Dr. Kelen's formulas omit the effect of the internal elastic work due to shear. These statements are misleading because Dr. Kelen actually

[10] Chf. Designer, Quinton, Code & Hill-Leeds & Barnard, Engrs., Consolidated, Los Angeles, Calif.

[10a] Received by the Secretary April 13, 1932.

[11] In "Die Staumauern," 1926, p. 44.

shows curves for the determination of the governing stresses; that is, at the abutment, both due to water load and dead load. No such "short-cut" is offered by the authors.

The introduction of the influence of shear is a refinement, but is more of academic than of practical value as far as circular, inclined arches of constant thickness are concerned. The allowable span of such arches is small as compared to the allowable span of pressure-line arches; that is, arches in which the center line coincides with the line of support in the statically determinate basic system.

To form an idea of the influence of shear, apply Dr. Kelen's formulas and diagrams to the authors' "Numerical Example" in the Appendix.

As $v = \frac{t}{l}$, the extreme fiber stresses at the abutment due to water load are:

$$f_w = \frac{P_1}{t} \pm \frac{6\,M_1}{t^2} = \frac{P_1}{v\,l} \pm \frac{6\,M_1}{v^2\,l^2}$$

or abbreviated,

$$f_w = [fw']\,w'l \qquad (79)$$

in which, l equals one-half the span.

Thus, for the stress at the extrados,

$$fw_e = [fw'_e]\,w'l \qquad (80a)$$

and for the stress at the intrados,

$$fw_i = [fw'_i]\,w'l \qquad (80b)$$

From Dr. Kelen's diagrams, which hold for the metric system only, $fw'_e = -\,14.30$; and $fw'_i = +\,17.20$; and from the authors' numerical example: $w' = 42.6$ lb. per cu. ft. $= 0.632$ metric ton per cu. m.; and, $l = r \sin 71 - 45 = 25.79 \times 0.9497 = 24.50$ ft. $= 7.47$ m.

Consequently,

$$\begin{aligned} fw_e &= -\,14.30 \times 0.682 \times 7.47 = -\,72.85 \text{ metric tons per sq. m.} \\ &= -\,7.53 \text{ tons per sq. ft.} \end{aligned}$$

and,

$$\begin{aligned} fw_i &= +\,17.20 \times 0.632 \times 7.47 = +\,87.60 \text{ metric tons per sq. m.} \\ &= +\,9.05 \text{ tons per sq. ft.} \end{aligned}$$

Reference to Table 1 for Elevation 140 and solving Equations (80):

$$\begin{aligned} fw_e &= \frac{P_1}{t} - \frac{6\,M_1}{t^2} = \frac{3\,990}{2.58} - \frac{6 \times 18\,470}{6.66} = -\,15\,083 \text{ lb. per sq. ft.} \\ &= -\,7.542 \text{ tons per sq. ft.} \end{aligned}$$

and,

$$fw_i = \frac{P_1}{t} + \frac{6\,M_1}{t^2} = +\,18\,177 \text{ lb. per sq. ft.} = +\,9.089 \text{ tons per sq. ft.}$$

Similarly, for dead load and Dr. Kelen's formulas:

$$fc_e = [fc'_e]\, w'_c\, l \quad \text{.......................} \quad (81a)$$

and,

$$fc_i = [fc'_i]\, w'_c\, l \quad \text{.......................} \quad (81b)$$

in which, $fc'_e = +\,3.85$; and $fc'_i = -\,0.90$.

From the authors' example, $w'_c = 102.50$ lb. per cu. ft. $= 1.637$ metric tons per cu. ft. Introducing these values in Equations (81),

$$\begin{aligned} fc_e &= +\,3.85 \times 1.637 \times 7.47 = 47.08 \text{ metric tons per sq. ft.} \\ &= +\,4.86 \text{ tons per sq. ft.} \end{aligned}$$

and,

$$\begin{aligned} fc_i &= -\,0.90 \times 1.637 \times 7.47 = -\,11.00 \text{ metric tons per sq. ft.} \\ &= -\,1.135 \text{ tons per sq. ft.} \end{aligned}$$

Based on Table 1 for Elevation 140:

$$fc_e = \frac{9\,475}{2.58} + \frac{6 \times 6\,605}{6.66} = +\,9\,618 \text{ lb. per sq. ft.} = +\,4.809 \text{ tons per sq. ft.}$$

and,

$$fc_i = \frac{9\,475}{2.58} - \frac{6 \times 6\,605}{6.66} = -\,2\,272 \text{ lb. per sq. ft.} = -\,1.136 \text{ tons per sq. ft.}$$

The foregoing shows that for practical purposes the two results are equivalent; that is, the work of deformation of shear in arches of short spans and large central angles, such as with ordinary multiple-arch dams, is small.

From an economic standpoint, the circular arch of constant thickness is not desirable for an inclined arch barrel, because the crown thickness is governed by the abutment thickness and thereby becomes over-dimensioned.

In all homogeneous structures it is desirable to keep the governing stresses uniform. The ideal condition naturally would be to have uniform stresses throughout such a structure, as suggested by Herman Schorer, Assoc. M. Am. Soc. C. E., for buttresses.[12] With arches, however, it is quite a different matter as, according to an old Chinese saying, "it [the arch] never sleeps." Arches, and particularly concrete arches under constant live load, are subject to continuous change in shape due to a change of temperature and relative humidity of the surrounding air.

Uniform strength, therefore, could only be temporarily reached in the case of a structure being designed as a pressure-line arch of uniform strength subject to compensating elastic thrusts due to live load, dead load, and to change in temperature and moisture, respectively. Use of this compensating feature has been made with long-span bridge arches in closing them at such a temperature that least stresses will result for a mean yearly temperature.

The difference in the stress distribution and the volume of an inclined pressure-line arch and a circular one may best be illustrated by an example. For a multiple-arch dam in Southern California, with a buttress spacing of 60 ft., a maximum height of 180 ft., more or less, and an inclination of the

[12] "The Buttressed Dam of Uniform Strength," *Proceedings*, Am. Soc. C. E., November, 1930, Papers and Discussions, p. 1947.

arch generator of 45°, the arches were designed as pressure-line arches (three-centered) and the elastic thrust, H_e, due to live load, dead load, and a change in temperature, was determined by the equation:

$$H_e = H_{eN} + H_{eT} = \frac{-\sum_0^{\phi_1} p \cos\phi \frac{\Delta s}{t} - \epsilon \Delta t l E}{\sum_0^{\phi_1} y^2 \Delta g + \sum_0^{\phi_1} \cos^2\phi \frac{\Delta s}{t} + 3 \sum_0^{\phi_1} \sin^2\phi \frac{\Delta s}{t}} \quad \text{. . (82)}$$

in which, for a 1-ft. arch slice, with the crown 169 ft. below the water surface:

Δg = elastic weight of element $= \frac{\Delta s}{I}$;
I = moment of inertia;
Δs = length of arch element = 3.355;
y = ordinates of center line of arch from x-axis through the elastic center;
ϵ = coefficient of thermal expansion = 0.000005;
Δt = change in temperature = $-5°$ Fahr.;
l = one-half span = 26.50 ft.;
E = modulus of elasticity = 2.50×10^6 lb. per sq. in.;
ϕ_1 = one-half central angle = 66° 30′.

The solution of Equation (82) is most conveniently arranged in tabular form and the detailed steps are not presented herein. For the stated conditions, H_e is found equal to $-$ 29 920 lb. The corresponding stresses at the crown are: $f_c = +418$ lb. per sq. in., and $f'_c = +167.40$ and -238.60; and the combined stresses are, $f''_c = +585.40$ and $+179.40$. Likewise, at the abutment (springing line), $f_s = +393$ lb. per sq. in.; and $f'_s = +321.0$ and -347.0; and the combined stresses are, $f''_s = +46.0$ and $+714.0$ lb. per sq. in.

This three-centered arch may be approximated by a circular arch of the following dimensions: $r = 29.40$ ft.; $r^2 = 865$; $t = 5.98$ ft. (based on equal volume); $\frac{t}{r} = 0.2033$; $r_e = 32.39$ ft.; $r_e^2 = 1\,050$; $\phi_1 = 65°\ 00'$; and $\alpha = 45°\ 00'$.

The head at the crown of the arch is 169 ft. and: $p_e = 169 \times 62.50 = 10\,565$ lb. per sq. ft.; $p_e r = 310\,500$; $p_e r^2 = 9\,133\,000$; $Q_0 = 0.076$; $\left(1 + \frac{t}{2r}\right) - Q_0 = 1.018$; $\left(1 + \frac{t}{2r}\right) - Q \cos\phi_1 = 1.065$; $Q_0 \left(1 - \frac{\sin\phi_1}{\phi_1}\right) = 0.0147$; $Q_0 \left(\frac{\sin\phi_1}{\phi_1} - \cos\phi_1\right) = 0.0293$; $w' = 62.50 \times 0.707 = 44.20$ lb. per cu. ft.; $w' r_e^2 = 46\,420$; and $w' r_e^2 r = 1\,365\,000$. Hence, $Z = 0.09$; $Z_1 = 0.102$; $Z_2 = 0.00515$; and, $Z_3 = 0.0165$.

Furthermore, $w'_c = 150 \times 0.707 = 106$ lb. per cu. ft.; $w'_c\, tr = 18\,650$; and $w'_c\, tr^2 = 548\,200$. Similarly, $Z' = 0.736$; $Z'_1 = 1.34$; $Z'_2 = 0.0274$; and, $Z'_3 = 0.012$.

Stresses at the crown are found to be as follows:

For Uniform Water Load:

$$P_0 = 1.018 \times 310\,500 = +\,316\,300 \text{ lb.};$$

and,

$$M_0 = -\,0.0147 \times 9\,133\,000 = -\,134\,250 \text{ ft-lb.}$$

For Variable Water Load:

$$P_0 = 0.09 \times 46\,420 = +\,4\,175 \text{ lb.}$$

and,

$$M_0 = +\,0.00515 \times 1\,365\,000 = +\,7\,030 \text{ ft-lb.}$$

For Dead Load:

$$P_0 = 0.736 \times 18\,650 = +\,13\,725 \text{ lb.}$$

and,

$$M_0 = -\,0.0274 \times 548\,200 = -\,15\,025 \text{ ft-lb.}$$

For Change in Temperature of $-\,5°$ Fahr.:

$$H = -\,0.000005 \times 5 \times 2.50 \times 10^6 \times 144 \times 5.98 \times 0.075 = -\,4\,040 \text{ lb.}$$

and,

$$M_0 = -\,4\,040 \times 29.40 \times 0.193 = -\,22\,930 \text{ ft-lb.}$$

The combined results are:

$$P_0 = +\,316\,300 + 4\,175 + 13\,725 - 4\,040 = +\,330\,160 \text{ lb.}$$

and,

$$M_0 = -\,134\,250 + 7\,030 - 15\,025 - 22\,930 = -\,165\,175 \text{ ft-lb.}$$

and the corresponding stresses are:

$$f''_c = +\frac{330\,160}{5.98 \times 144} \pm \frac{6 \times 165\,175}{5.98^2 \times 144} = \begin{cases} +\,576.0 \text{ lb. per sq. in. (extrados)} \\ +\,191.0 \text{ lb. per sq. in. (intrados)} \end{cases}$$

The stresses at the abutment are found to be, as follows:

For Uniform Water Load:

$$P_1 = +\,1.065 \times 310\,500 = +\,330\,700 \text{ lb.}$$

and,

$$M_1 = +\,0.0293 \times 9\,133\,000 = +\,267\,700 \text{ ft-lb.}$$

For Variable Water Load:

$$P_1 = +\,0{,}102 \times 46\,420 = +\,4\,737 \text{ lb.}$$

and,

$$M_1 = +\,0.0165 \times 1\,365\,000 = +\,22\,530 \text{ ft-lb.}$$

For Dead Load:

$$P_1 = +\,1.34 \times 18\,650 = +\,25\,000 \text{ lb.}$$

and,

$$M_1 = -\,0.012 \times 548\,200 = -\,6\,575 \text{ ft-lb.}$$

For Temperature Change of $-\,5°$ Fahr.:

$$H_1 = -\,4\,040 \times 0.4226 = -\,1\,710 \text{ lb.}$$

and,

$$M_1 = +\,0.385 \times 117\,300 = +\,45\,200 \text{ ft-lb.}$$

The combined results are:

$$P_1 = + 330\,700 \times 4\,737 \times 25\,000 - 1\,710 = + 358\,727 \text{ lb.}$$

and,

$$M_1 = + 267\,700 + 22\,530 - 6\,575 + 45\,200 = + 328\,855 \text{ ft-lb.}$$

and, the corresponding stresses are:

$$f''_s = + \frac{358\,727}{5.98 \times 144} \pm \frac{6 \times 328\,855}{5.98^2 \times 144} = \begin{cases} + 34 \text{ lb. per sq. in. (extrados)} \\ + 800 \text{ lb. per sq. in. (intrados)} \end{cases}$$

Thus, with the circular arch, the maximum crown stress is 2% smaller than the maximum crown stress of the pressure-line arch of equal volume, but the maximum governing abutment stress of the circular arch is 12% greater than the corresponding stress of the pressure-line arch. This shows that geometrical shapes of structures which lend themselves beautifully to mathematical analysis are not necessarily economical.

For an inclination of the arch generator of between 45° and 50° and a central angle between 130° and 145°, which holds true for ordinary multiple-arch dams, the economic limit of buttress spacing for circular arches is 40 ft., more or less, and about 400 ft. for the pressure-line arch.

LARS R. JORGENSEN,[13] M. AM. SOC. C. E. (by letter).[13a]—Perhaps the most complete analysis of the stresses in inclined arches of multiple-arch dams, to be found in the English language, is that presented in this paper. Taken together with recent papers on the investigation of stresses in buttresses and their distribution (Holmes,[14] Schorer,[15] Jakobsen,[16] and Noetzli[17]), the stress analysis of the complete multiple-arch dam is now in a satisfactory stage of completion.

Not the least valuable contribution of the authors is the large amount of work put into the graphical representation of necessary coefficients. All the final formulas for moments, thrust, and shear would be very lengthy except for the coefficient, z, which once for all has been calculated by the authors and the results put into the form of curves for all conditions most likely to occur in ordinary design and construction work. The curves are drawn to such scales that values of z can be read from them with great enough accuracy to use for all cases. For even greater accuracy, photostat copies of the original curves, to a larger scale, are available, as stated in the paper.

Since all the final formulas consist of only two to three factors besides the coefficient, z, a slide-rule can be used, whereas if z could not be taken from curves, but had to be computed every time, a calculating machine would be absolutely necessary and the total time consumed in calculating the stresses in an arch of a multiple-arch dam would be days instead of hours.

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[13] Cons. Hydro-Elec. Engr., Berkeley, Calif.

[13a] Received by the Secretary April 15, 1932.

[14] "Determination of Principal Stresses in Buttresses and Gravity Dams," by W. H. Holmes, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., January, 1932, p. 71.

[15] "The Buttressed Dam of Uniform Strength," by Herman Schorer, Assoc. M. Am. Soc. C. E., *Loc. cit.*, November, 1930, Papers and Discussions, p. 1947.

[16] "Stresses in Gravity Dams by Principle of Least Work," by B. F. Jakobsen, *Loc. cit.*, September, 1930, Papers and Discussions, p. 1613.

[17] "Improved Type of Multiple-Arch Dam," by Fred A. Noetzli, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 342.

The material is well presented. At the close of the paper, it is stated that the unit stresses in the concrete and steel are not considered in the paper since such information can be found in many handbooks. For the sake of completeness it would be helpful if the authors indicated in their closure the desirability of increasing the arch thickness near and at the abutments and gave an approximate solution for finding the new location of the neutral axis.

The authors recommend reinforcing steel in both faces. It is evident that steel is required in both faces at the abutments; but it is not so evident that reinforcing steel is required along the up-stream face at the crown. A rise in temperature with reservoir empty could be assumed compensated for by the shrinkage and permanent rib-shortening as far as tensile stresses in the up-stream face at the crown are concerned.

The writer has noticed at times, that wave action on a multiple-arch dam has the effect of creating a temporary higher water level at the abutments than at the crown, say, a maximum of 5 ft. The wave will climb the **V** perhaps at the time the valley reaches the crown. This would be of no importance with the water low in the reservoir, but with the water close to the crest, the phenomenon may call for steel in the up-stream face near the crest, if the arch is not provided with a stiffening rib or cornice.

The paper is of much practical value for the designer of multiple-arch dams and thanks are due the authors for their work.

FRED A. NOETZLI,[18] M. AM. SOC. C. E. (by letter).[18a]—Considerable painstaking labor was involved in developing the formulas, computing the coefficients, and platting the numerous diagrams given in this paper. The diagrams will be helpful in the design of multiple-arch dams. As mentioned by the authors, curves serving a similar purpose have been previously published by Dr. N. Kelen,[19] who did not, however, consider it necessary to include the effect of the internal elastic work due to shear. Diagrams by means of which the stresses from uniform water pressure can be obtained directly from the "cylinder" stresses, have also been published.[20] The corresponding thrusts and moments can be readily computed, if desired, from the equation,

$$f_c^{u,d} = \frac{N}{t} \pm \frac{6\,M}{t^2} \qquad \text{(83)}$$

in which, f_c^u and f_c^d are the stresses, in pounds per square foot, at the up-stream and at the down-stream faces, respectively.

In a number of the earliest multiple-arch dams the central angle of the arches is 120 degrees. The application of the elastic theory to such arches indicated that a larger center angle is desirable to keep the arch stresses within permissible limits. In the authors' example, the central angle of the arches is about 145 degrees. For long arch spans and large central angles the influence of the dead load and of the variable water-load becomes increasingly important.

[18] Cons. Hydr. Engr., Los Angeles, Calif.

[18a] Received by the Secretary June 14, 1932.

[19] "Die Staumauern," Berlin, 1926.

[20] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 539.

The paper gives a complete and theoretically exact method of designing the arches of multiple-arch dams. Further improvements in the art are to be looked for rather from the point of view of construction than from that of design.

W. J. STICH,[21] ASSOC. M. AM. SOC. C. E. (by letter).[21a]—This paper graphically presents data by which an exact solution of the stresses in the inclined uniformly thick circular arches of multiple-arch dams becomes a relatively simple matter.

The tremendous amount of labor involved in the preparation of a paper of this nature can perhaps only be fully realized by the authors themselves, and much commendation is due them for the convenient form in which the coefficients and formulas are presented. The writer can appreciate this fact, to some extent, because he undertook to check the first sixty odd formulas developed in connection with obtaining the coefficients for moment, thrust, and shear due to variable water load, weight of arch, temperature, and uniform water load for arches with fixed ends.

In order to determine the increase in tensile stress developed and, therefore, the additional reinforcement required, when the variable water load and weight component are included in the analysis of the arch stresses, a calculation was made by the writer for an arch ring practically at the crest of Dam No. 1 as given by the authors in the numerical example in the Appendix. The critical arch ring selected, will serve to indicate the degree of consideration merited by the non-uniform loads (weight component and variable water load) acting on the arch.

In conformity with the basic data given by the authors for Dam No. 1, the following physical data were selected by the writer for the arch ring at Elevation 195 (abutment): $r_e = 26.50$ ft.; $t = 2.00$ ft.; $\frac{t}{r} = 0.079$; $2\,\phi_1 = 143.0°$; and temperature drop = 20 degrees. The head at the arch crown is: $(208—195) — 26.50 \cos 47° \times \text{versin } 71°\ 30' = 0.66$ ft., and $p_e = 0.66 \times 62.5 = 41.25$ lb. per sq. ft.

By means of the authors' curves and using the foregoing physical data the summary of moments, thrusts, and shears in Table 5 was obtained for Elevation 195.

Analysis of Table 5 indicates that under the conditions assumed a critical tensile stress of 233 lb. per sq. in. is developed at the abutment extrados of the arch ring in question. By means of the standard formulas for the design

TABLE 5.—COMPUTATION MOMENT, THRUSTS, AND SHEARS FOR ELEVATION 195

Loading	r_e, in feet	t, in feet	$\frac{t}{r}$	$2\phi_1$, in degrees	P_0, in pounds	P_1, in pounds	M_0, in foot-pounds	M_1, in foot-pounds	S_1, in pounds
Uniform water.......	26.50	2.00	0.079	143.0	1 086	1 090	—59	110	2.7
Variable water.......	26.50	2.00	0.079	143.0	3 380	3 800	6 338	17 400	5 050
Dead load...........	26.50	2.00	0.079	143.0	3 982	7 450	—2 570	—5 330	—1 725
20° temperature drop.	26.50	2.00	0.079	143.0	—1 268	—402	—7 590	14 210	1 202

[21] Civ. Engr., Los Angeles City Board of Education, Los Angeles, Calif.

[21a] Received by the Secretary August 11, 1932.

of reinforced concrete arches subject to bending and direct stress, the longitudinal steel required to reinforce the arch ring adequately is then fixed as 1-in. square bars at 18-in. centers for both up-stream and down-stream faces.

In order to investigate the influence of the non-uniform loads (variable water load and weight of arch) upon the critical stresses developed and upon the amount of reinforcement required, a second analysis of the arch ring at Elevation 195 was made by the writer. In this instance, curves based on the formulas developed by the late William Cain, M. Am. Soc. C. E., in his paper entitled "The Circular Arch Under Normal Loads,"[22] were used to determine the crown and abutment stresses.

By means of these curves and with the further assumptions of a 20° temperature drop and a uniform water load on the arch corresponding to the average head between the crown and the abutment, a critical tensile stress of 109 lb. per sq. in. was developed at the abutment extrados for Elevation 195. Assuming in this case that all the tension is to be absorbed by the steel, $\frac{3}{4}$-in. round bars, at 14-in. centers, would be required in each face. This spacing agrees with the arbitrary minimum as utilized in present-day practice.

The results of this investigation would indicate that an increase in reinforcing steel of approximately 80% is required when the effect of the non-uniform loads is included in analyzing the arch ring stresses of multiple-arch dams. It should be noted, however, that this increase applies only to the upper arches and down to a level of approximately 100 ft. below the crest. At elevations lower than this, the bending moments induced by the variable water load and weight component become relatively small; and, furthermore, the weight component tends to equalize the load distribution on the arch.

Unfortunately, a more economical distribution of material and stresses is obtained in the design of multiple-arch dams when the normal arch is made thicker at the abutments than at the crown. This increase in thickness may be attained advantageously by making the intrados of the arch three-centered. In this case, the arch stresses developed at the abutments will be less critical than those indicated by applying coefficients such as those introduced by the authors, which are based on the analysis of a circular arch of constant thickness. A more exact method of analysis would dictate the use of the elastic theory wherein the non-uniformity of the load due to the slope of the arch barrel, as well as the variation in thickness of the arch, are given due consideration.

[22] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233.

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DISCUSSIONS

THE COMPENSATED ARCH DAM

Discussion

BY MESSRS. R. A. SUTHERLAND, LARS R. JORGENSEN, L. T. EVANS, AND H. W. SIBERT

R. A. SUTHERLAND,[2] ASSOC. M. SOC. C. E. (by letter).[2a]—This paper is a valuable contribution to the knowledge of the subject of arch dams, and the author is to be congratulated on his candid presentation of a very unusual proposal for improving the loading conditions of such dams. No one who has given thought to the subject doubts that the design of arch dams can be and will be improved, but it is recognized that the number of unknown factors is so large that a perfect knowledge of load distribution will probably never be achieved. While model tests are undoubtedly helpful, the writer believes that they should be interpreted qualitatively rather than quantitatively; that is, as a guide toward a better understanding of load distribution rather than as a means of making exact deductions from the model as to the behavior of the full-sized dam.

The writer is glad that Mr. Karpov has shown the importance of considering the shape of the dam site, because this has a considerable effect, as far as can be known, in determining the relative distribution of load as between arch and beam at the different sections.

With regard to the proposal that V-shaped notches should be provided, the writer is of opinion that any removal of material from the arch must, in practice, reduce the share of load carried by it and increase that taken by beam action. Thus, if the design is based on a given distribution of load as between arch and beam, and the notches are then provided ostensibly to obtain uniform stress conditions in the arch, the result will be that the arch, in deflecting to close the notches (which must occur before it can take any appreciable share of load) will throw an undue load on the beams and falsify the assumptions on which the design was made. At the same time, twisting actions will be caused in the beam sections, which will probably cause secondary stresses just as pernicious as the mal-distribution of stress which it is aimed to avoid.

NOTE.—The paper by A. V. Karpov, M. Am. Soc. C. E., was published in April, 1932, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[2] Wellington, New Zealand.

[2a] Received by the Secretary May 10, 1932.

Apart from all this, the writer does not believe that it is possible to obtain uniform stress distribution in an arch by removing material therefrom in any manner. The gaps must first close before the arch takes load, and when they are closed, the arch is in just as bad case as a solid arch, with the further disadvantage of having its shear strength almost destroyed. Pre-stressing, as advocated by Dr. Vogt, appears on theoretical grounds to be the only method by which uniform stresses in the loaded arch can be obtained. It may be urged that the closing of the gaps that are proposed actually results in pre-stressing the arch in the desired manner, but in practice the heavy concentrations of load, spalling of the concrete, etc., will destroy any nicely calculated effect of this kind.

Some approach to a better distribution of arch stresses can be obtained by varying the curvature to suit the loads, and the writer would look to developments in this direction for improvement in the practical design of arch dams.

LARS R. JORGENSEN,[3] M. AM. SOC. C. E. (by letter).[3a]—The author has proposed the design of an arch dam, in which the thrust at the abutments is to be applied tangent to the neutral axis of each arch ring of uniform thickness and in which single arch elements can be shifted automatically into the correct position under load to allow the thrust due to full load to be uniformly distributed over the arch sections along the arch ring. In such a case there would be no bending moment, which would be desirable.

Mr. Karpov has done considerable work on this design; he is working in a field in which there are still wide gaps and his contribution is valuable in helping to supply improvements.

In Part III, he states:

"The problem of finding the proper shape of the arch resolves itself into finding the equation of the center-line curve of the arch for which the changing radial loads applied at the up-stream face will produce zero bending moments at every point of the center-line curve."

The greatest problem is to find the value of the changing radial loads on the arch at various points, as upon the accuracy of this depends the adequacy of the resulting design. As the author has stated, the cantilever carries most of the water load at the bottom, and at the top the arch generally has to carry a part of the cantilever load, in addition to all the water load.

It is well known that in order to compute the percentage of load taken by the various cantilevers several assumptions must be made, that are rather difficult to formulate correctly. Therefore, they are often omitted—especially the water-soaking effect and the difference in value of E between wet and dry concrete. Whether or not there will ever be actual vertical tension in the up-stream face of the cantilevers of an arch dam depends mainly upon the degree of water-soaking effect present in the section, but also to quite an extent on the difference in the amount of the total deformation of wet and dry concrete. The total deformation of dry concrete is approximately 1.6

[3] Cons. Hydro-Elec. Engr., Berkeley, Calif.

[3a] Received by the Secretary May 31, 1932.

times that of wet concrete. This fact will cause more load to be thrown on the arch than if, as is customary, E is assumed constant.

The water-soaking effect has thus far been omitted, when calculating the variable radial load on the arch slice, because less is known about it than any other factor. At its worst—that is, when the distribution varies from a maximum at the wet up-stream face to zero at the dry down-stream face—the arch may be forced to hold the cantilever back in addition to supporting all the load. At its best the water-soaking effect can be omitted. When designing an important structure the worst condition that can happen is of more interest than anything less than that. If the worst condition of loading is assumed, however, the working stresses can safely be chosen rather high.

The Diablo Dam on the Skagit River, in Washington, was designed to carry all the load on the arch without help from the cantilever. The section could have been made thinner (16 ft. at the crest and 98 ft. minimum at the bottom) had the cantilever action been taken into consideration and the possible water-soaking effect omitted, but it was assumed that the two actions would about equalize one another.

The crushing strength of the mass concrete in this dam at the age of one year was about 6 000 lb. per sq. in. It would appear to be safe to use maximum stresses, or at least maximum apparent stresses, of 1 000 lb. per sq. in. for such concrete in large dams, when assuming all the load as taken by the arch, since in such case the actual maximum stress should be less, whereas had the effect of the cantilever been included as supporting part of the load and had the water-soaking effect been omitted, the actual maximum stress certainly would be higher. Of course, material can always be saved by using higher stresses. If the cantilever is considered as taking a certain proportion of the load, using a constant E, omitting the water-soaking effect, and allowing 600 lb. per sq. in. for maximum compression, the actual stresses may well be about 1 000 lb. per sq. in. The water-soaking effect is always favorable to arch action and unfavorable toward forcing the cantilever to take load.

Concrete with a crushing strength of 6 000 lb. per sq. in., when one year old, will safely withstand a maximum calculated stress of 1 000 lb. per sq. in. To secure good concrete is much more important than to obtain mere low stress in a high dam since such concrete is also tight, and the life of a high concrete dam is in direct proportion to its tightness.

At one time the writer thought that bending moments in an arch dam could be eliminated just by changing the shape of the arch, making it rounder near the abutments, where the center line of pressure bends in a down-stream direction. This can not be done, however; it does require dividing the arch ring and shaping the voussoirs or blocks as shown in the author's Fig. 6, in order to allow the arch to adjust itself. More blocks will allow a finer adjustment than just a few blocks. Making the blocks 40 to 50 ft. long, as between ordinary contraction joints, would not accomplish the necessary adjustment and it bcomes a pertinent practical question, therefore, how far to go with the division of the ring in order to avoid the bending.

The cost of constructing these blocks accurately and making them water-tight must at least balance the cost of taking care of the bending by means of added material at and near the abutments, where this added material incidentally takes care of the maximum shear also. The shearing strength of the concrete is an important item for holding the dam in place especially toward the bottom.

Considering the uncertainties in computing the variable radial loads on the arch elements and the many loose joints in the arch, the compensated arch should be dimensioned conservatively, because some bending may actually be thrown upon it.

Mr. Karpov, who is connected with a large corporation having first-class facilities for testing purposes, is in a favorable position for carrying on investigations of the greatest unknown influential factor in this connection, the general distribution of the water-soaking effect.

A study of the behavior of round or square horizontal concrete columns, subject to water pressure at one end and to the atmosphere at the other end, should yield valuable information about the distribution of the water-soaking effect throughout a body of concrete. It would be necessary to paint the sides of the specimens with a heavy coat of asphaltum to prevent evaporation and also to provide metal plugs at intervals for measuring purposes.

The writer believes that time and money spent on such an investigation would be worth while and suggests that the author go into this matter with the same determination as he has shown in his previous work. Before much faith can be placed on the positive existence of cantilever support, more information must be available on the distribution of water-soaking effect than now exists.

L. T. EVANS,[4] JUN. AM. SOC. C. E. (by letter).[4a]—Without a doubt, Mr. Karpov has made a valuable addition to engineering literature. For the most part his paper seems theoretically correct except for the statement that only arches of constant thickness can have zero moment at any point along the arch axis, neglecting the effect of rib-shortening. It is to be regretted that Mr. Karpov made this error because constant thickness is not necessary in order to have zero moment along the arch axis.

In checking over Mr. Karpov's work, equation by equation, there is no objection to be raised until the derivation of Equation (19) is reached. Every step is mathematically legitimate and no errors are involved. From the mathematician's standpoint all is well, but not from an engineer's standpoint. The engineer is interested in Equations (5), (8), and (11).

Consider a circle defined by the equation, $(x')^2_1 + (y')^2_1 = (R')^2$; from which, $\frac{dy'_1}{dx'_1} = -\frac{x'_1}{y'_1}$. Next, shift the origin so that $x'_1 = x_1 - x'$, and $y'_1 = y_1 - y'$. It follows that:

$$\frac{dy'}{dx'} = -\frac{x_1 - x'}{y_1 - y'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (92)$$

[4] Long Beach, Calif.

[4a] Received by the Secretary August 19, 1932.

Equation (92) is at once recognized as a transposed form of Equation (19). Since Equation (19) is purely a statement of mathematics and can be derived by mathematical operations entirely free from all reference to an arch, it is obvious that it does not imply that the moment on the axis of an arch must equal zero. In other words, Mr. Karpov has lost all trace of his moment, shear, and thrust by the time he has reached Equation (19). As no moment, shear, or thrust terms are added in any operation between Equation (19) and Equation (28), it is impossible to see how any results obtained by any expression between these equations can be said to follow from the fact that the moment on the arch axis must equal zero.

Correct methods of attack have been devised, that eliminate the possibility of such errors. One method is to evaluate Equations (8) and (11) and set the sum equal to zero. Equation (8) can be expressed in such a manner that the thickness of the arch rib at any point appears as a variable. It will then be possible to study the effect of a variable thickness on the moments along the arch axis.

If the origin of co-ordinates is taken at the "crown" of the arch on the axis of the arch, it is possible to set up an expression for the moment about the variable point, x, y, which is:

$$M_{xy} = \frac{p'}{2}\left[x^2 + \left(y + \frac{t_0}{2}\right)^2 - \frac{t_x^2}{4}\right] + M_0 + V_0 x - H_0 y \quad \ldots\ldots (93)$$

In order to satisfy Equation (5):

$$\frac{p'}{2}\left[x^2 + \left(y + \frac{t_0}{2}\right)^2 - \frac{t_x^2}{4}\right] + V_0 x - H_0 y = 0 \quad \ldots\ldots\ldots\ldots (94)$$

It is seen that Equation (94) can be satisfied by an infinite number of values of t_x, and as it is the equation of the neutral axis of the arch, it is obvious that there can be many shapes of arch ring that satisfy the conditions imposed by Equation (5). When the variable, t_x, is expressed in terms of x, y, and t_0, the equation of the neutral axis has a definite form.

Again, consider the problem in the light of the column analogy[6], as presented by Hardy Cross, M. Am. Soc. C. E. The change in moment will be given by the expression:

$$m_1 = \frac{P}{A} + \frac{M'_x}{I'_x}\, x + \frac{M'_y}{I'_y}\, y \quad \ldots\ldots\ldots\ldots\ldots\ldots (95)$$

and since it is assumed that there is to be no moment on the arch axis, the value of P, M'_x, and M'_y, must be equal to zero. As these factors are zero, the change in moment is also equal to zero and Equation (5) is satisfied independently of A, I'_x, and I'_y in Equation (95). As these terms are dependent upon the variation of the thickness, t_x, this is an undisputable proof that the condition of zero moment along the arch axis does not define the variation of the thickness of the rib.

It might also be mentioned that the moment at any point along the arch axis is equal to the product of the thrust at that point and the eccentricity.

[6] See *Bulletin No. 215*, Eng. Experiment Station, Univ. of Illinois.

As long as the section of the rib is made symmetrical about the line of thrust, there is no moment, irrespective of the amount of section on each side of the pressure line.

This error detracts little from the value of Mr. Karpov's work; yet the writer feels that attention should be called to it for the benefit of engineering students who might use the paper as a reference.

H. W. SIBERT,[6] ESQ. (by letter).[6a]—In Part III of Mr. Karpov's paper, Equation (28) is incorrect. Hence, his conclusion that the thickness must be constant is erroneous. This failure to define p' has made it difficult to locate the source of his error. At the beginning of Part III, one gets the impression that p' is the radial load, a term which presupposes a circular arc; while immediately after Equation (4), p' is called the external load. In the discussion that follows, p' is considered to be the normal pressure of the water per unit area on the up-stream face at any point of the horizontal section

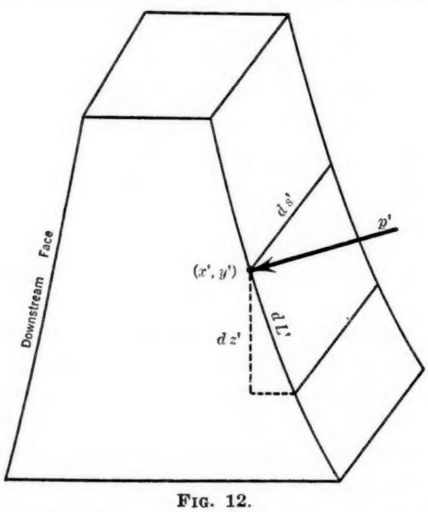

FIG. 12.

of the dam shown in Fig. 3. Since p' is the pressure perpendicular to the up-stream face, it depends only upon the depth of the horizontal section below the surface of the water. Hence, p' is independent of x' and y'.

Consider an infinitesimally small area on the up-stream face having a length, dL', and a width, ds' (see Fig. 12). The normal force, dP', on this area is:

$$dP' = p' \; ds' \; dL' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

If dz' is the vertical projection of dL', the horizontal component of this normal force is:

$$dP'_h = \frac{dz'}{dL'} dP' = p' \; ds' \; dz' \quad \ldots\ldots\ldots\ldots\ldots\ldots (97)$$

[6] Associate Prof. of Mathematics, Coll. of Eng. and Commerce, Univ. of Cincinnati, Cincinnati, Ohio.

[6a] Received by the Secretary August 1, 1932.

Thus, the horizontal component of the force per unit of height is:

$$dQ'_h = \frac{d P'_h}{dz'} = p' \ ds' \ldots\ldots\ldots\ldots\ldots\ldots\ldots (98)$$

Furthermore, from Fig. 3 of the paper, the clockwise moment of dQ'_h about the point, (x_1, y_1), is:

$$dm' = (x_1 - x') \frac{dx'}{ds'} dQ'_h + (y_1 - y') \frac{dy'}{ds'} dQ'_h \ldots\ldots\ldots\ldots (99)$$

Hence, by use of Equation (98):

$$dm' = p' (x_1 - x') dx' + p' (y_1 - y') dy' \ldots\ldots\ldots\ldots (100)$$

Finally, since $M_1 = \int \frac{dm'}{p}$, the identical value of M_1 given in Equation (8) of Mr. Karpov's paper is obtained.

Since not only p', but also x_1 and y_1 are independent of x' and y', the evaluation of the two integrals given in Equation (8) is an easy matter. Thus,

$$M_1 = \left[\frac{p'}{2} (x_1 - x')^2\right]_{-\alpha'_m}^{x'_1} - \frac{p'}{2} (y_1 - y')^2\Big]_{o'}^{y'_1} \ldots\ldots\ldots (101)$$

or,

$$M_1 = \frac{p'}{2}\left[- (x_1 - x'_1)^2 + (x_1 + \alpha'_m)^2 - (y_1 - y'_1)^2 + (y_1 - o')^2\right] . (102)$$

As the square of the hypotenuse equals the sum of the squares of the other two sides, $(x_1 - x'_1)^2 + (y_1 - y'_1)^2 = \frac{t^2_1}{4}$. The substitution of this result in Equation (102) gives the following value for M_1:

$$M_1 = \frac{p'}{2}\left[(x_1 + \alpha'_m)^2 + (y_1 - o')^2 - \frac{t^2_1}{4}\right] \ldots\ldots\ldots\ldots (103)$$

When this value of M_1 and the value of M_2 given in Equation (11) are substituted in Equation (5), the condition for zero moment at each section becomes,

$$\frac{p'}{2}\left[(x_1 + \alpha'_m)^2 + (y_1 - o')^2 - \frac{t^2_1}{4}\right] + (H_0 - H_R) y_1 - V_0 (\alpha_m + x)_1 = 0 \ldots (104)$$

As (x_1, y_1) is any point on the neutral axis, less confusion will result if this point is replaced by the variable point (x, y), and, at the same time, t_1 is replaced by t. Thus, the condition for zero moment at every section is,

$$\frac{p'}{2}\left[(x + \alpha'_m)^2 + (y_1 - o')^2 - \frac{t^2}{4}\right] + (H_0 - H_R) y - V_0 (\alpha_m + x) = 0 \ldots (105)$$

For the particular case, in which $t = k$ (a constant), the neutral axis is the circular arc:

$$\frac{p'}{2}\left[(x + \alpha'_m)^2 + (y - o')^2 - \frac{k^2}{4}\right] + (H_0 - H_R) y - V_0 (\alpha_m + x) = 0 \ldots (106)$$

Moreover, if $t = 2(y - o')$, the neutral axis is the parabolic arc:

$$\frac{p'}{2}(x + \alpha'_m)^2 + (H_0 - H_R)\,y - V_0(\alpha_m + x) = 0 \qquad (107)$$

Furthermore, if $t = k + cx$ (k and c being constants and the numerical value of c being less than 2), the neutral axis is the elliptical arc:

$$\frac{p'}{2}\left[(x + \alpha'_m)^2 + (y - o')^2 - \frac{(k + cx)^2}{4}\right] + (H_0 - H_R)\,y - V_0(\alpha_m + x) = 0 \qquad (108)$$

Equation (105) shows that t can be any function of x and y. However, for each particular function of x and y assigned to t, the equation of the neutral axis is of a particular form, as has been demonstrated in the preceding paragraph.

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DISCUSSIONS

DEVELOPMENT OF HYDRO-ELECTRIC POWER AS AN AID TO IRRIGATION

Discussion

BY MESSRS. A. H. MARKWART, AND R. V. MEIKLE

A. H. MARKWART,[2] M. AM. SOC. C. E. (by letter).[2a]—The writer was much impressed with Mr. Cragin's paper. The significant point gained from it is that irrigation power has had its golden opportunity in Arizona. Obviously, the basic factor in that situation is the high cost of fuel.

The value of irrigation power is, fundamentally, the value of the fuel saved in equivalent steam plants, since practically every irrigation power development requires a kilowatt of steam capacity for each kilowatt of irrigation hydro-electric power. This becomes necessary by reason of the general stipulation that irrigation draft shall govern the release of water. Thus, power demand, when not synchronized with irrigation draft, must be met by supplemental steam. To summarize, the cost of fuel alone, less transmission costs, sets the value of irrigation hydro-electric power, up to the point where all hydro-electric power on a load-carrying basis rather than on an irrigation draft basis, is cheaper than all steam-electric power at load centers.

If fuel is high in price, the irrigation power development will have the advantage because its investment is low, this being limited to the generating equipment, the penstock, and perhaps a short conduit. The dam, the reservoir, and other parts normally required for developing head in a power project, are here necessary to and chargeable to the irrigation project.

In general, irrigation power can be absorbed most readily in a regional system having a relatively high annual load factor, such as the systems in Central and Southern California. Irrigation and drainage pumping are, of course, ideal as irrigation power loads. A low load-factor system is not helpful in absorbing irrigation hydro-electric power.

In California at present (1930), irrigation power is suffering from the presence of enormous quantities of cheap fuel, both oil and natural gas.

NOTE.—The paper by Charles C. Cragin, M. Am. Soc. C. E., was presented at the meeting of the Irrigation Division, Sacramento, Calif., April 24, 1930, and published in April, 1932, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[2] Vice-Pres., Pacific Gas & Elec. Co., San Francisco, Calif.

[2a] Received by the Secretary May 11, 1932.

While fuel oil delivered in Arizona might cost $1 per bbl. for transportation only, in addition to the value of the oil, in California there are great quantities of natural gas available and going to waste, and which should be conserved as is any other natural resource. Under these circumstances, it will be somewhat more difficult for irrigation power to do as well in the future as it has in the past. While the falling waters of an irrigation project should be conserved, as a natural resource, the natural resources in gas, which unlike water have no possibility of being replenished, should likewise be protected.

R. V. MEIKLE,[3] M. AM. SOC. C. E. (by letter).[3a]—The benefits to be derived by the development of hydro-electric power in connection with irrigation storage have been clearly presented in this paper. In his analysis of the subject, the author's conclusions are generally applicable to the power development by irrigation districts on the Stanislaus, Tuolumne, and Merced Rivers in the San Joaquin Valley of California where the total installed capacity is 120 000 h. p. and the output is about 390 000 000 kw-hr. yearly. The power from these California developments is disposed of in a different manner for each project. One project receives storage water for irrigation free, in exchange for its power rights to the power company; another project sells the entire output wholesale under a 40-year contract; the third project combines distribution within its area and wholesale delivery to the power company under long-term contracts. In each instance, the development of hydro-electric power has been a substantial aid to irrigation.

The sale of power generated at these irrigation storage developments to the power company under a long-term contract is undoubtedly the safest and easiest method of disposing of the power output.

It is possible for an irrigation enterprise to engage in the general power business successfully, but this step should not be taken unless it is impossible to make a reasonable and fair agreement for the sale of the output, to the local power company. In an irrigation enterprise, the taxpayers should receive the first consideration in any matter that affects the financial structure of their project. The distribution of power by the irrigation project may lead to the paralleling of lines and destructive competition with the local power company. Reduction of power rates on the project resulting from competition or low rates made to please the voters, may not benefit the taxpayer to the extent that he would be benefited by the income that could be derived from the sale of the power to the power company without the risks involved in power distribution. In the California district where power is distributed at very low rates, the taxpayers represent only 60% of the power customers.

The question of future power distribution by irrigation districts is becoming of less importance in California, due to the inability of the districts to finance such expensive undertakings and to the fact that the power companies have been co-operating with the irrigation districts to the fullest extent

[3] Chf. Engr., Turlock Irrig. Dist., Turlock, Calif.

[3a] Received by the Secretary May 31, 1932.

possible in the handling of the power output from the district projects. Furthermore, lower domestic rates may be expected from the power companies. To increase the use of electricity, the rates must be low enough for the average family or customer to take advantage of the many uses of electric power.

The lowest domestic power rate under the irrigation district distribution, of the projects mentioned is, as follows: 15 kw-hr., $1.00; next 15 kw-hr., $0.05; next 50 kw-hr., $0.025; next 120 kw-hr., $0.02; and more than 200 kw-hr., $0.01. This rate, under the conditions of rural distribution, is possibly too low, but no money has been lost in supplying the 5 000 customers under the rate, and the growth of business, in kilowatt-hours, notwithstanding the present depression (1932), has continued at the rate of 4% yearly.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

IMPROVEMENT OF HUDSON RIVER BY NARROWING THE NAVIGABLE FAIRWAY

Discussion

BY MESSRS. WILLIAM B. MOSS, LYTLE BROWN, AND H. A. MARMER

WILLIAM B. MOSS,[3] ASSOC. M. AM. SOC. C. E. (by letter).[3a]—Considered hydraulically, the author's opening statement is undoubtedly sound. The continued application of the narrowing process would probably benefit the hydraulic regimen; but that it would improve the Hudson River for navigation is questionable. Improvement, from the standpoint of the port engineer, must co-ordinate and, to an extent, subordinate hydraulic principles with, and to, the demands of safe navigation and harbor accommodation. A wide channel, easily maintained at the maximum usable depth, is generally much better than a narrow and deeper fairway.

The Lower Hudson River has a variety of important commercial servitudes. In its lower part it is practically a sub-port lined on both shores with berths for the largest merchant ships in deep sea, coastal, and river traffic. Its volume of cross-traffic and the congestion existing at times are too well known to need description. Above the bend at West 23d Street large areas are given over to the anchorage of vessels, including naval anchorages for capital ships, in the deep channel adjacent to the New York shore north of West 72d Street. Just below the river bed, are important tunnels, protected by comparatively thin layers of mud and clay from possible serious injury through the sinking of a vessel and/or an explosion of its cargo in close proximity to the shells of the tubes. Extra channel width is an important consideration as a safety factor in case of such a contingency. The river is the thoroughfare for vessels and flotilla tows bound to and from the distant up-river port of Albany and other river points. It passes on to these northern river areas a tidal movement of the progressive wave type, as well as the great tidal stream, with extensive hydraulic reactions.

Prior to 1890 harbor lines were established by State and city authorities; since that date they have been fixed by the Federal authorities. Owing to

NOTE.—The paper by Lynne J. Bevan, M. Am. Soc. C. E., was published in May, 1932, *Proceedings*, Part 1. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[3] Engr., New York Harbor Line Board, U. S. War Dept., New York, N. Y.

[3a] Received by the Secretary June 21, 1932.

the conflict of interest mentioned by the author, it is probable that not all the subsequent modifications of these lines have had the sole purpose of improving the river for the benefit of navigation. The points fixing the present least width between pierhead lines on the New Jersey and New York shores at West 23d Street, Manhattan, were established by pier lines adopted by the Riparian Board of New Jersey in 1865 on the one shore, and by the City of New York in 1871, on the other.

In recommending Federal harbor lines in 1890, the first New York Harbor Line Board stated[4] its opinion that harbor lines should have been drawn tangent to Castle Point, N. J., since it projects so far beyond the general shore line. However, since such a modification would have interfered with vested rights, duly acquired under the State laws of New Jersey, the Board recommended the adoption of lines fixed by the laws of New York and New Jersey at that time. An exception was made in the pierhead line between the Battery and West 23d Street, New York City, which was extended outward to accommodate navigation. The maximum extension was about 155 ft. at Canal Street. According to this report the original width of the river at West 14th Street had been materially impaired by artificial encroachments on both banks; and this result was deplored because, originally, the rocky headland known as Castle Point had formed a marked gorge which should not have been made worse by artificial shore extensions. These statements indicate that this first Federal Harbor Line Board decided, against its inclination and judgment, to allow encroachments to continue, which had been legalized before the War Department received authority to regulate these matters.

Pre-war current observations and data appeared to support the opinion that the velocity of the river current had increased. New and comprehensive data were published[5] in 1925 by the U. S. Coast and Geodetic Survey in the pamphlet, entitled "Tides and Currents in New York Harbor," by H. A. Marmer, M. Am. Soc. C. E., which contains tabulations of tidal current observations from records of the Survey between 1854 and 1922. These data do not indicate any large percentage of change in the maximum tidal velocities of the Lower Hudson River during the period covered. Tables 63 and 64 of the pamphlet indicate observed ebb-current velocities at several stations and at the several periods reading between 2.5 and 2.8 knots per hour and an observation in 1919 giving a velocity of 3.11 knots, all reduced to normal tide values. The latter observation, which is No. 58 in Tables 63 and 64, is discredited, as it is out of line with other observations in the vicinity made in 1922 (Nos. 57 and 59), although it resulted from two days' observations. Its location off Fort Washington Point, where practically no contraction in the width of the stream has occurred, eliminates it from consideration in discussion of the author's subject. It is mentioned only as it indicates the familiar and disconcerting manner in which an individual observation frequently breaks up the continuity of a chain of data, and raises an element of

[4] Rept., U. S. Corps of Engrs., 1890, pp. 816 and 821.

[5] *Special Publication No. 111,* U. S. Coast and Geodetic Survey.

doubt. Observations in the tables numbered 2, 16, 19, and 55 of the pamphlet are also difficult to reconcile with others in their vicinities.

In Table 64,[6] the more recent observations Nos. 1, 8, 10, 13, 18, 24, 26, 29, 34, 42, 49, 57, and 63, include both pole and meter readings for approximately the same depths, which differ from 5 to 25% from each other. It seems to the writer, in view of these discrepancies, and the extreme difficulty in obtaining accurate current velocity measurements mentioned by the author (as well as the absence of information as to the depth of the earlier observations) that there is little to warrant a definite conclusion that the strength of the tidal currents has either decreased or increased noticeably since 1854. A conclusion that it has not increased nor decreased largely is believed to be justified by the observations.

Meter observations of the ebb current at strength were made by the U. S. Engineers on December 26 and 27, 1912, which showed maximum velocities of nearly 6 ft. per sec., or $3\frac{1}{2}$ knots per hour. On these dates, lunar phase produced spring tides which were recorded as about $1\frac{1}{2}$ ft. above normal. Reduced to normal tide these observed velocities would be about 2.7 knots, which agree fairly well with the compiled maximum velocities of the Coast and Geodetic Survey pamphlet. These current observations were taken at intervals of 5 ft. in depth in the thread of the swiftest current at three stations off Barclay Street, Chelsea Piers and 52d Street, Manhattan, respectively. They agreed with the Coast Survey determinations in showing a fairly regular decrease in current with depth at two stations, but, in the Chelsea Section, a nearly uniform velocity was observed in 1912 between the 10 and 50-ft. depths, the average rate of flow being from 2.1 to 2.2 knots, reduced to normal tidal range. The effect of the tidal currents on deep-draft vessels will be proportional to the mean velocity of the vertical section they displace. Normally, this velocity will be considerably greater than near-surface flows at flood strength. The velocity near the bottom will be the controlling factor in erosion by tidal current.

The paper by Mr. Bevan exhibits quite a collection of curves and tabular data which deal with changes in the cross-sectional areas of the lower river at intervals and which were originally compiled principally from U. S. Coast and Geodetic Survey charts. Close examination reveals inaccuracies in assumption and basic data, which weaken the conclusions apparently reached. Errors in assumption are: (1) That the effective cross-section is included between the pierhead lines; and (2) that the charts used show soundings of the date of publication. Errors in data include: (1) Using charts with insufficient soundings; (2) not considering dredging that increased a cross-section; and (3) excluding free areas shoreward of the pierhead line not occupied by structures at the given dates and sections. Critical study has been made of Sections 3, 6, 12, and 15, shown by the author's Figs. 3 to 6, inclusive, as well as Fig. 2. For the purpose the writer used Coast Survey charts of 1930 issue. To these were transferred soundings of 1855 taken from office tracings of two charts entitled: "U. S. Coast Survey, A. D. Bache, Superintendent,

[6] *Special Publication No. 111*, U. S. Coast and Geodetic Survey, p. 132.

New York Bay and Harbor Surveys in 1855, Copied for the Commissioners for the Preservation and Protection of the Harbor of New York from Encroachments." They are Hydrographic Sheets Nos. 477 and 496 (scale 1: 10 000).

Table 1 and Fig. 2 of Mr. Bevan's paper show a fairly uniform deepening across channel at the Chelsea Pier Section between 1845 and 1855, the increase of section being 10 500 sq. ft. About one-half this amount was lost by the advance of the shore prior to 1855. At this early period a pier extended outshore to about the position of the present pierhead line and caused an additional obstruction beyond the pier line of 1855. Figs. 2 and 4 show a pierhead line of 1914 about 100 ft. channelward of the 1897 pierhead line, which is an error, as the 1897 line remains the established line. In this section shoaling is indicated along the New York shore between 1855 and 1874, followed by deepening in 1912. Between these dates the Chelsea Piers were built and the shoal area in front of them was dredged more than 40 ft. deep, and 997 000 cu. yd. of material were removed outside the pierhead line on a frontage of about 3 400 ft., with an average width of about 500 ft. This deepening would result in increases of cross-section of 6 000 and 9 000 sq. ft., the lower value probably applying to the place of the section.

In the sections shown, only a few widely separated soundings were obtained from the older maps. As a result the bottom contours are long straight lines with only three to four breaks across channel. If more frequent soundings are taken from the original hydrographic sheets and the entire clear portion of the river between existing structures is included, the results show considerably less increase of section between 1855 and 1930. It must be pointed out that many soundings on the Coast Survey chart of 1930 are quite old. Revised depths are given on the chart only wherever the U. S. Engineers have dredged or where they have made surveys in more recent years. Thus, on Chart No. 369.8, nearly all soundings northerly of West 158th Street date back to the survey of 1885-86, and many of those in the deeper water south of this point are of the same date. Since the soundings available and the harbor lines were the same at Fort Washington Point both in 1910 and 1930, the areas of cross-sections should be practically the same for both dates, but the author's Tables 3 and 4 show a difference of 15 200 sq. ft. as of these dates, or nearly 10 per cent. Even on Coast Survey Chart No. 369.4, issued in 1930, through the deeper sections, where untouched by dredging or not covered by a later survey, a few of the original soundings of 1885-86 are still retained. Most of the older soundings, however, date from 1899 to 1910. Thus, in the vicinity of Section 3 (Fig. 3), due to much dredging, out of fifty-three soundings only two refer back to the 1885 survey and seven to the 1899-1910 survey; but at Section 6 (Fig. 4), out of a total of forty-three, three are from 1885 and twenty-four from 1899-1910; while, at Section 9 (for which no diagram is shown), out of forty soundings, six date from 1885 and four from 1899-1910. The so-called 1930 data, therefore, refer to mixed dates and not to the year 1930.

Referring to the section numbers of Tables 3 and 4, the amount of advancement of the physical shore by structures or fill in various sections is approximately as follows:

Section	Shore advance, in feet	Section	Shore advance, in feet
2	600	10	1 500
3	800	11	1 000
4	1 500	12	700
5	1 600	13	600
6	270–320	14	500
7	1 500	15	500
8	1 200	16–18	0
9	1 100		

It will be seen that Table 3 shows a decrease, or a slight increase, in area in most of the sections where the largest contraction of stream has occurred, notably at Sections 3, 4, 5, 10, 11, and 12.

The writer has made comparison of the river sections from the Coast Survey Charts of 1855 and 1930 at the author's Sections 3, 6, 12, and 15. Two tiers (three in Section 6) of reference squares have been drawn on the charts across the river at each section, and the approximate amount of deepening or shoaling in the 75-year period has been estimated. These data are shown by Table 6, the average change in depth, in feet, for each lettered square of each tier being given by figures. A preceding plus sign indicates deepening, and a minus sign, shoaling.

TABLE 6.—Estimated Deepening or Shoaling in Sections 3, 6, 12, and 15

Tier	Description	Reference Grid Letters								
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
					Section 3: Fulton Street					
North..	Total change....	—10	+9	+2	+3	+7	+3	±0	+1	±0
	Dredging.......	.....	+9	+2	.....	+4.5	+1.5	.....	.....	.....
South..	Total change....	—5	+6	+1	+3	+7	+2	—1	+3	±0
	Dredging.......	.....	+6	+0.5	+1	+6	+1	.....	.....	.....
					Section 6: Chelsea Section					
North..	Total change....	+7	+5	+8	+8	+2	+5	.....	.....	.....
	Dredging.......	+2	.....	.....	.....	+2	+5	.....	.....	.....
Middle.	Total change....	+8	+4	+5	+4	+3	+5	.....	.....	.....
	Dredging.......	+2	.....	.....	.....	+3	+5	.....	.....	.....
South..	Total change....	+5	+3	+1	—1	+1	+6	.....	.....	.....
	Dredging.......	+3	.....	.....	.....	+1	+6	.....	.....	.....
					Section 12: West 89th Street					
North..	Total change....	—10	—1	+3	+4	+4	+5	+2	—20	.....
	Dredging.......	.....	.....	+3	.....	.....	.....	.....	.....	.....
South..	Total change....	—10	+2	+5	+7	+4	+4	+2	—15	.....
	Dredging.......	.....	.....	+4	.....	.....	.....	.....	.....	.....
					Section 15: West 138th Street					
North..	Total change....	+1	+4	+3	+1½	+7	+9	—3	.....	.....
	Dredging.......	+1	+1½	.....	.....	.....	.....	.....	.....	.....
South..	Total change....	—1	+7	+4	+4	+4	+4	—12	.....	.....
	Dredging.......	.....	+3	.....	.....	.....	.....	.....	.....	.....

An examination of Table 6 shows clearly a deepening of considerable areas. A striking difference is indicated between results in the north and south tiers at Section 6, and it is obvious that there is less increase in section across the lower tier than the upper. Thus it was found that by drawing the section line across the river about opposite 6th Street, Hoboken, N. J., the areas would be approximately 155 000 sq. ft. for the 1855 soundings, and 150 000 sq. ft. for the 1930 map, the latter including about 5% of dredging.

Taking into consideration the actual physical shores as limited by fill and structures at the dates indicated, the writer has computed the areas of the four cross-sections, in square feet, using for the 1930 sections the Coast Survey charts of the 1930 issue (see Table 7).

TABLE 7.—AREAS OF SECTIONS 3, 6, 12, AND 15, IN SQUARE FEET

Section	1855	Natural, 1930	Dredging	Total 1930
3	163 000	149 000	7 500	156 500
6	149 000	151 000	8 000	159 000
12	131 000	117 500	500	118 000
15	128 000	121 000	5 000	126 000

A recomputation of the 1855 cross-section at Fort Washington Point (Section 18), gives an area of about 145 000 sq. ft. The great depth at this point, the impossibility of accurate development of the bottom because of the small scale of the maps, and the infrequency of soundings accentuate the unreliability of the results.

That there has been deepening of parts of the channel is evident from the comparison of the earlier and latest soundings. That it is due to erosion of the bottom, which is known to be generally quite soft, is readily deduced. In general, the greatest increase in depth has been in the deeper areas, close to the thread of deepest water, which apparently has not shifted materially in the past seventy-five years. Some of this deepening may be attributed to the disturbance of the bottom by large vessels, maneuvering to and from docks along shore. This disturbance is very active when a vessel's powerful screws are reversed in checking speed, or when the vessel is started ahead or astern. Unquestionably a large increase by erosion beyond a depth of about 45 ft., involving large quantities of material, is not desirable from the standpoint of improvement, because the eroded material will be deposited in some other part of the channel or harbor, and will add to the cost of dredging in other places.

Although much larger sectional areas occur up stream and down stream from the Holland Tunnel, it is interesting to observe that near or along the line of the crossing (Canal Street), there is now (1932) an average depth of only 42 ft. and a section between pier ends of only 133 000 sq. ft. West of the middle the river has been dredged an average of about $3\frac{1}{2}$ ft., which increased the section about 7 000 sq. ft. If this section is dredged from pierhead line to pierhead line to an average depth of 45 ft., the effective area will then be about 140 000 sq. ft. along the tunnel line. This would leave about 16 ft. of cover over the tunnel.

A study of the maps and surveys indicates that at times there has been slight shoaling on the New Jersey shore in the vicinity of the Delaware, Lackawanna and Western Railroad Ferry at Jersey City, N. J., and on the New York shore between 19th and 30th Streets. A recent development is the shoaling along the east shore between West 75th Street and West 152d Street, Manhattan, which is probably due largely to the fill made in extending Riverside Park. It may be probable that a tidal stream, particularly on the ebb, has the tendency to concentrate its energy along the line of deepest water, where higher velocities will occur, than it had before the deepening and concentration took place. Ambrose Channel, which was cut through the bar at the entrance to New York Harbor and deepened from 40 to 45 ft., now has tidal velocities only slightly less than those of the Lower Hudson River. However, they are considerably higher than were formerly observed in the adjacent shoaler channels that have been superseded by Ambrose Channel, although it is probable that the cross-section of the mouth of the Lower Bay is larger than at that former period.

In the absence of more reliable data and better interpretation of those at hand, it is believed that the conclusions reached and the implications suggested by Mr. Bevan are far from decisive. Even if the paper had demonstrated conclusively that the river's hydraulic regimen had been improved and its current velocities reduced, there is great uncertainty as to whether or not this hypothetical effect could be attributed solely to the narrowing caused by advancing the shore and pier lines. Other factors should be considered. Minor changes in the ranges and heights of tides produce marked current changes. Channel deepening and widening both in the Lower Hudson and in other parts of the harbor and its tributaries and entrances, and large changes in the volume of the tidal prism over the long period under consideration, probably have had an induced or direct effect upon the tidal reaction and currents. There is no question that the author's suggested method of "improvement" by narrowing solely to secure better hydraulic conditions, would injure rather than improve the Hudson River for navigation, even if it were not largely infeasible because of the existence of such barriers as the vehicular and railroad tunnels.

LYTLE BROWN,[7] M. AM. SOC. C. E. (by letter).[7a]—Data are presented in this paper, purporting to show that the successive encroachments of the piers into the fairway of the Hudson River in New York Harbor, have increased the cross-sectional area and decreased the tidal current velocities. Years ago apprehension was expressed by engineers of high standing that the contraction of the river by pier encroachment would produce an undesirable increase in tidal velocities. Experience has shown that the apprehension was not well founded. Whether such increase in cross-section as may have occurred, should be ascribed to the contraction or to other causes is an open question. Aside from the dredging done by the Federal Government, scour from ships' propellers may be a material factor. This waterway is extensively used by the

[7] Maj.-Gen., U. S. Army; Chf. of Engrs., U. S. Army, Washington, D. C.

[7a] Received by the Secretary June 30, 1932.

largest vessels afloat. Their propellers are not far from the bottom when they pass through the channel, and the strong currents set up have a pronounced action in scouring the soft material in the river bottom.

The part of New York Harbor formed by the Hudson River is perhaps the most important artery of commerce in the world. It is well known that the commerce of the Port of New York far exceeds that of any other port of any nation, and the Hudson River Channel is the heart of the port. The number of vessel trips recorded in the Hudson River Channel exceeds 1 200 000 per annum. The commerce in a normal year, exclusive of car-ferry traffic and cargoes in transit, is about 40 000 000 tons, with a value of approximately $7 000 000 000. More than 100 000 000 persons are transported yearly over the channel. The cost of maintaining the channel averaged $125 000 per year during the five years, from 1927 to 1932. It is obvious that the exigencies of commerce completely overshadow any engineering problem of maintaining the depths in the harbor.

A comprehensive study of collisions in New York Harbor shows that during the five years, 1926 to 1930, inclusive, 118 collisions occurred in the Hudson River Channel, 6 vessels being sunk by collision. The average annual damages from collision in this section of the harbor exceed $300 000.

These figures show why it is that the War Department, in the administration of the laws for the protection and preservation of the navigable waters of the United States, is not inclined to view with favor the proposals advanced from time to time to further constrict the width of this great avenue of commerce.

H. A. Marmer,[8] M. Am. Soc. C. E. (by letter).[8a]—In this paper four theses are put forward by the author: (1) That successive encroachments on the navigable fairway of the Lower Hudson have set in action natural forces which have scoured the river bottom; (2) that this scouring has been of such magnitude as to more than counterbalance the restriction in width, having actually increased the cross-sectional area of the waterway; (3) that the range of the tide in the river has remained practically constant, in spite of the extension of the pierhead line into the channel of the river; and (4) that the velocity of the tidal currents has actually decreased.

Taking up each of these in turn, the author's comprehensive analysis of the changes of the cross-sectional areas in various sections of the Lower Hudson, as presented in Tables 3 and 4, clearly proves the first thesis. This analysis brings out the fact that an increase in the depth of the river has taken place coincident with the advance of the pierhead lines into the channel.

These tables at the same time show that the increase in depth has more than compensated for the restriction in width due to the extension of the pierhead line into the channel. As between 1855 and 1930, Table 3 shows that the average cross-sectional area has increased from 130 590 sq. ft. to 137 010 sq. ft., an increase of 5 per cent. Of this increase only 2 560 sq. ft., or 2%, was due to dredging. The second of the author's theses, namely, that

[8] Asst. Chf., Div. of Tides and Currents, U. S. Coast and Geodetic Survey, Washington, D. C.

[8a] Received by the Secretary August 11, 1932.

the scouring has been of such magnitude as to have increased the average cross-sectional area of the Lower Hudson, is thus established. It is to be noted, however, that this increase in cross-sectional area is an average increase, and that for seven of the eighteen sections, the area now is less than in 1855, in spite of the dredging that has been done in these sections.

The third thesis, that relative to range of tide, presents a much more difficult question than that centering in the change of area of cross-section, because the latter can be determined readily from the charts showing the hydrographic features of the river at different dates. However, the range of the tide at any place varies from day to day, from month to month, and from year to year. Quite apart from the effects of wind, weather, and fresh-water run-off, the variations in range go through various cycles, with periods up to nineteen years. It follows, therefore, that two series of observations made at different times at the same place will give different ranges of tide, unless these observations in each case cover a period of nineteen years.

In connection with most engineering operations, it is impractical to observe tides over periods of nineteen years. Instead, observations are made over much shorter periods of time—from a month to a year—and the results derived from these observations are reduced to mean values.

On the open coast, or in large coastal bodies of water, the range of the tide from a given series of observations may be reduced to a mean value either by comparison with simultaneous observations at some primary tide station, or by the use of a factor, depending on the longitude of the moon's node. Investigation proves that even from so short a series of observations as a month the value of the mean range can generally be determined correctly within 0.1 ft.

For tides in tidal rivers, however, the determination of the mean range is much more difficult than on the open coast or in large bays or sounds because, in addition to the fluctuations in range due to purely tidal causes, there are the fluctuations arising from the variations in fresh-water run-off. In streams that carry the drainage waters from large areas, the variations in range due to the fluctuations in fresh-water flow are frequently much greater than those due to purely tidal causes.

It is obvious, therefore, that the determination of the mean range of tide at any point in a tidal river from a short series of observations is far from a simple matter. Theoretically, it is possible to correct the range for the fluctuations in fresh-water flow; but practically this is not feasible for the simple reason that the data requisite for this are not at hand.

In the Hudson River, the fresh-water flow, as a rule, is greatest in the spring months and least in the summer and fall months. The true tidal range, therefore, can be determined more accurately from observations covering the summer and fall months, when the disturbing effects of the fresh-water flow are least. In Table 5 the author gives the mean range of the tide at Yonkers, N. Y., for a number of years, as determined from observations covering the three months of July, August, and September. This table illustrates clearly that in a tidal river the range of the tide derived from several months of observations and reduced to a mean value by correction

for the longitude of the moon's node, will exhibit fluctuations of 0.1, or even, 0.2 ft., from year to year.

Such fluctuations in the values of the mean range are to be ascribed to difficulties involved in the observations themselves, and to fluctuations arising from the effects of wind, weather, and fresh-water run-off. It follows, therefore, that if from observations at any point in the Hudson River a difference in the mean range of as much as 0.1 or 0.2 ft. is found, as between two years, it cannot be concluded that this difference indicates a change in the range at that place. The author concludes that the data of Table 5 are indicative of the constancy of the range at Yonkers.

While the range for any one year in Table 5 cannot be considered as giving the mean range to better than 0.1 ft., the mean value for a group of three years is obviously a much closer approximation to the mean value than any one year. If from the data of Table 5 averages are formed for 3-year groups, the following values are derived:

1910–12	3.31 ft.	1921–23	3.49 ft.
1913–15	3.35 ft.	1924–26	3.49 ft.
1916–18	3.39 ft.	1927–29	3.55 ft.

It is a striking fact that each of the values for the 3-year groups shows an increase in range from 1910 to 1929, the difference between 1910–12 and 1927–29 being 0.24 ft. This is altogether too large to be ascribed to such fluctuations as characterize the means from year to year. The conclusion is inescapable that, on the basis of the data in Table 5, there has been an increase in range of tide at Yonkers between 1910 and 1929, of about 0.25 ft.

If the range of tide at Yonkers derived from the more recent observations, is compared with the earlier observations cited by the author in Table 5, it appears that there has been a decrease from about 4.0 ft. in the Fifties, and $3\frac{3}{4}$ ft. in 1898, to the present value of 3.55 ft. To evaluate, properly, the effects of the extension of the pierhead line on the range of the tide, it will be necessary to consider these changes in range in greater detail—the decrease to 1910 and the subsequent increase.

The fourth thesis in the paper, that relating to the current, offers greater difficulties than the thesis relating to the tide. As the author states, current measurements involve numerous difficulties. In addition, the current at any point changes from day to day. In part, this change is of a periodic nature, similar to that in tides, and, in part, it is of a non-periodic nature, due to fluctuations in wind, weather, and fresh-water run-off. A further complication arises from the fact that the current in any cross-section varies from point to point both in horizontal and vertical directions.

To determine whether or not there has been any change in the tidal current in a river, from short series of observations made at different times, is thus not a simple matter. To begin with, the observations must be made at the same point, in order that differences due to differences in position may not be introduced; and, in the second place, the results must be reduced to mean values. This is done most conveniently by taking the changes in the current to be proportional to the changes in range of the tide.

The current observations that have been made in the Hudson River, have been primarily for the purposes of the mariner, and to bring out the local peculiarities at different points. These observations, therefore, do not lend themselves readily to determining whether or not there has been a change in the velocity of the current consequent on the extension of the pierhead line into the channel.

The author concludes that the mean tidal current velocity must have decreased as a result of the pierhead extension into the Hudson, inasmuch as the cross-sectional areas have increased, while the range of the tide has remained substantially constant. This is a logical deduction in so far as the tidal current alone is concerned; but the Hudson River also serves as a channel for the drainage waters from a large territory. These drainage waters set constantly seaward, giving rise to a non-tidal current, which lessens the flood current and increases the ebb current.

Since the drainage waters are less dense than the sea water brought in by the tide they tend to remain on the surface. This is evidenced by the fact that, in the Hudson River, the velocity of the ebb current is greater than the flood near the surface, while below mid-depth the flood current is equal or frequently even greater than the ebb. A decrease in the width of the channel, therefore, will decrease, in effect, the cross-sectional area of the channel for the drainage waters and give rise to an increased velocity for the ebb current.

In assembling and discussing the data bearing on this problem of the improvement of the Hudson River, the author has brought forward an important engineering problem which merits much more research than has been given it previously in this country.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

APPLICATION OF DURATION CURVES TO HYDRO-ELECTRIC STUDIES

Discussion

BY MESSRS. PAUL F. KRUSE, DONALD H. MATTERN, JAMES E. STEWART, DON JOHNSTONE, L. T. GUY, AND JOEL D. JUSTIN.

PAUL F. KRUSE,[3] M. AM. SOC. C. E. (by letter).[3a]—This paper presents an interesting application of the graphic method of analyzing certain hydro-electric problems commonly known as the "duration curve method." Instead of plotting the duration graphs in terms of water flow and time, the authors go a step farther and plot them in terms of power flow or power rate and time.

What appears to the writer as one of the chief advantages of this procedure over the older flow-duration curve method for studies, such as those depicted by the authors, relates to the factor of head. The power capacity or rate available at any water-power site is a function of the product of the quantity of water flowing and the head through which it falls. In the use of the ordinary water-flow duration curves in studies of the combined operation of two or more plants on different streams, or at different sites on the same stream, the head available at each site must be taken into account and separate determinations must be made to obtain the power capacities and the energy that can be secured from the combined development.

By translating stream flow into power rate at each site and plotting this against time, thus taking into account the factor of head before plotting the graphs (as has been done by the authors), such power-output duration curves for two or more sites may be directly applied one to another for the purposes cited of obtaining the combined energy output of the two or more plants under consideration and the capacities required in each plant for such combined operation. The ordinates of the output-duration curve being in units of power and the abscissas in units of time, the area under the curve represents energy, and the area of any one diagram may be transferred or applied to any other diagram of the same scale.

NOTE.—The paper by G. H. Hickox and G. O. Wessenauer, Juniors, Am. Soc. C. E., was published in May, 1932, *Proceedings,* Part 1. This discussion is printed in *Proceedings,* in order that the views expressed may be brought before all members for further discussion.

[3] With Sanderson & Porter, New York, N. Y.

[3a] Received by the Secretary May 25, 1932.

The term, "flow duration curves," in the authors' statements of "Advantages," is believed to have been used erroneously. As stated in the paper under Case 1, "flow duration curves are not used directly for determining output rate and plant capacity; hence, they need not be constructed." The authors' procedure is sufficiently novel and noteworthy to warrant the use of terminology consistent with the analysis which expresses or implies the advantage relating to the factor of head, and to this end it is suggested that it may well become known as the "output duration curve," as used elsewhere in the paper by the authors, or more specifically the "power rate-duration curve" method of analysis.

DONALD H. MATTERN,[4] JUN. AM. SOC. C. E. (by letter).[4a]—The authors are to be commended for their presentation of this new application of duration curves. Their examples ably demonstrate its great versatility under suitable conditions, that is, where the streams under consideration have similar flow characteristics. It is thought advisable to emphasize this point because, to use the method on widely separated streams, some of flashy and others of more regular characteristics, having different flow distributions, would introduce conditions that could not be handled satisfactorily. It is obvious that two streams which have their heavy flow periods occurring several months apart could have duration curves that were similar in shape, and yet proper utilization of the water could not be accomplished by the method suggested by the authors.

It would be interesting to learn why "output in kilowatts" was used as ordinates in all their work instead of "cubic feet per second." It seems that to use the basic factor would aid in keeping the fundamental object in mind, namely, utilization of the water flowing in the stream. Use of "cubic feet per second" would eliminate the reduced effective heads that result at high-water periods and, in the case of a plant at a storage reservoir, when depletion or filling is in progress. Changing unit efficiencies under different head and operating conditions would also then be eliminated, with a resulting increase of accuracy and usefulness of final results.

It is true that this method is used for preliminary purposes only, but the work necessary to secure these benefits entails no additional effort. The writer has used both types of ordinates in somewhat similar studies and with those dealing with plant operation, and he has come to the conclusion that use of "cubic feet per second" gives more accurate and understandable results.

JAMES E. STEWART,[5] M. AM. SOC. C. E. (by letter).[5a]—Graphical analysis is the preferable method of studying engineering problems when the results are of approximately the same accuracy as those obtained by more laborious computation methods, and the writer believes that this paper marks a notable advance in the application of graphical analysis to hydro-electric studies.

After the method outlined by Messrs. Hickox and Wessenauer has been thoroughly mastered, users of it should be able to obtain approximately the

[4] Tyrone, Pa.

[4a] Received by the Secretary July 23, 1932.

[5] Hydr. Engr., West Virginia Power & Transmission Co., Pittsburgh, Pa.

[5a] Received by the Secretary July 25, 1932.

same results as by the use of the hydrograph (computation) method, with the great advantage of having such results in much more satisfactory form and accomplishing the work in about one-fourth the time necessary with the hydrograph method, aided by the fastest computing machines.

Checking time is reduced as much as the computing time, and where results are needed before complete checks on computations can be made, the results obtained by the duration curve method, on the average, can be depended upon to be more nearly free of error than when carried out by the hydrograph method. This is due to the practically self-checking features of the duration curve method.

It is believed that any hydro-electric engineer who thoroughly masters the duration curve method will never return to the hydrograph method, except perhaps for an occasional "spot" check. The difficulty in bringing about the universal use of the duration curve method for the type of hydro-electric problems outlined in the paper, does not lie in the method or in any particular difficulty in learning it. Rather the difficulty lies first, in human nature which is inherently slow to adopt anything new no matter how meritorious it may be; and, second, to the fact that unless the studies are extensive there will be no gain in total time due to the time used in learning the duration curve method. Nevertheless, the writer believes that it is only a matter of time until the duration curve method will displace the more laborious older methods wherever it is applicable.

Don Johnstone,[6] Jun. Am. Soc. C. E. (by letter).[6a]—That the authors have presented a method of hydro-electric investigation entailing less work than the old hydrograph method is scarcely to be questioned. The severe limitations imposed by two of the basic assumptions, however, reduce its usefulness considerably, and the writer believes that even in those cases to which it is applicable, a study by means of mass curves will yield the same results with less effort, as no re-arrangement of flows out of chronological order is involved.

The assumption that the streams under consideration have the same distribution of flow should be discussed further. The authors mention that the hydrographs of the tributaries of the Cheat River are "strikingly similar" to those of the main stream; yet no two streams have exactly similar hydrographs, and it is becoming increasingly important, with the development of systems involving plants located 100 miles or more apart, to be able to study, quickly and simply, hydro-electric possibilities on streams with widely varying hydrographs. Just how much variation in characteristics can take place before the method becomes inapplicable? It may be mentioned here that a system study by means of the mass curve makes allowance for any variation of flow distribution which may exist between the various plants, however great it may be.

The assumption which causes the greatest limitation of usefulness, however, is the third, requiring "that the storage available at each reservoir" be

[6] Surveyman, U. S. Engr. Office, Kansas City, Mo.

[6a] Received by the Secretary August 11, 1932.

"unlimited." The writer's experience has been that it is a rare case in which sufficient storage is available to regulate completely the flow of the stream. Still more rare is the case in which it would be economical to use that amount of storage. In general, topographic features set definite limits on the amount of available storage, and a method which permits of starting from this ordinarily known point (as does the mass-curve method) has a far wider range of application than one which entails a basic assumption which is so often unwarranted.

Furthermore, it should be emphasized that, having assumed unlimited storage at a reservoir plant, it is necessary to make a "stab in the dark" in selecting the average effective head to be applied to its discharge. The selection may be so far from the actual value that a repetition of the study will be almost imperative; when the study is begun with a definite assumption concerning storage, however, the average head—and hence the power rate—can be arrived at very closely in the first trial, as will be shown later.

The method as presented by the authors may permit of some simplification. For example, in determining the storage reservoir capacity in Case 1, considerable unnecessary work has been done. As the total demand on storage (represented in Fig. 3 by the area, *MD'N*) is known, and the date of maximum draw-down is determined from a mass curve, there is no apparent reason, in the example given, for re-arranging flows in order of magnitude. All that is actually required is to add the inflows occurring between full reservoir and the point of maximum draw-down, and subtract their sum from the total demand on storage.

As it is apparent from the authors' paper that the applicability of mass-curve methods to system studies is not generally appreciated, and as the statement has been made that such methods are applicable to all the cases covered in their examples, the writer feels that it is not out of place in this discussion to describe them in sufficient detail to support his contention.

The common Rippl mass-curve serves for the study of a single plant. When system operation is involved, the "mass-curve of equivalent flow" must be introduced. Its construction is best explained by an example.

Assume a system of three hydro-electric plants located along a single stream. Let the actual (natural) flow at Plant No. 1 (in acre-feet per month) be designated Q_1, the simultaneous discharge at the second plant by Q_2, and at the lowest plant by Q_3. The power which could be produced by this natural flow is proportional to,

$$Q_1 (H_1 + H_2 + H_2) + (Q_2 - Q_1) (H_2 + H_3 + (Q_3 - Q_2) H_3$$

in which, H_1, H_2, and H_3 are the average operating heads at the various plants. Now, compute the hypothetical flow at the up-stream plant, which would be required to produce the same amount of power if no down-stream inflows occurred. This quantity, designated Q_e, is, therefore,

$$Q_1 + \frac{(Q_2 - Q_1)(H_2 + H_3)}{(H_1 + H_2 + H_3)} + \frac{(Q_3 - Q_2) H_3}{(H_1 + H_2 + H_3)}$$

In other words, for each month, Q_e is the flow at the up-stream plant, plus the incremental inflow at each plant down stream therefrom, multiplied

by the ratio of the head through which that increment can act to the total head in the system. The mass curve of equivalent flow is plotted from the successive summation of Q_e.

As the curve has been computed in acre-feet, storages can be applied to it as to an ordinary mass curve, and there may be one reservoir in the system, or more than one. It is only necessary to remember that the equivalent storages must be used; that is, the storage capacity of each reservoir must be corrected by the same factors which were applied previously to the flows. Then, suppose a reservoir of 100 000 acre-ft. capacity to be located at the uppermost plant, and another of equal size at the second plant. If one-half the developable head of the stream occurs at the up-stream plant, then the water in the second reservoir is the equivalent of only 50 000 acre-ft. at the upper plant, and the total equivalent storage capacity of the system is 150 000 acre-ft.

The mean equivalent flow to which the stream can be regulated can be determined in the same manner as the mean flow from the ordinary mass curve. The total power (prime) is the product of this flow, the total average head of the system, and the proper factors for conversion to kilowatts.

The demand on equivalent storage in any month is obviously the difference between the mean equivalent flow to which the stream is to be regulated and the equivalent flow for the particular month. The storage requirement is then divided among the various reservoirs according to some fixed rule. In the example already introduced, suppose it has been determined that equal quantities should be drafted from the two reservoirs, and that in a certain month 10 000 acre-ft. of equivalent storage is necessary to the maintenance of prime. Then, if Q is the actual quantity drawn from each reservoir, $Q + \frac{1}{2}Q = 10\,000$, and Q is 6 667 acre-ft. (Note that 6 667 acre-ft. drawn from the lower reservoir has an "equivalent" value of 3 333 acre-ft., since it acts through only one-half the total head of the system.)

The next step in the study is to correct the natural flow at each plant by the draft or storage from all reservoirs up stream therefrom. The required installation at each plant is determined from the maximum regulated flow computed to occur at that point during a draw-down period. (The maximums at the various plants are not necessarily simultaneous.) Secondary output can also be calculated from the tables of regulated flow.

All the information required has thus been found by using a general method unhampered by the requirements of similarity of flow and unlimited storage, and without the necessity of going through the tedious work of constructing duration curves.

A claim for the author's method is that when the work is completed a good graphical representation is obtained; but although it may be good from an engineering viewpoint, it is likely to be unintelligible to the layman. As the purpose of a study of this nature is primarily to interest persons other than engineers in a project, the study—not only the results, but the method as well—should be made easily comprehensible; and it should be recognized definitely that few curves are more abstruse and confusing than duration curves. There is a lay tendency (unconscious, perhaps, but perhaps, there-

fore, all the more firm), to look upon abscissas as representing time in chronological order—and it requires a specialized study to understand graphs which are not prepared in this manner. On the other hand, both the mass curve and the hydrograph have the characteristic abscissas which make them readily understandable to the layman; the mass-curve study is direct, requires no repetition of trials, and its results can be transferred to a hydrograph presentation if that is desired.

The effect of head variation at reservoir plants was omitted from the authors' treatise as a matter of detail. It would be interesting, however, to know how they propose to handle it. As far as the writer can see, the only way to bring head variation into account is to make an analytical study on at least a monthly mean flow basis. It becomes quite important, then, if an analytical study is out of the question, to make the proper assumption as regards average effective head to be applied to reservoir plants studied by duration curves or mass diagrams. Any error in this assumption introduces the same percentage of error in the calculation of prime rate.

The average head during the critical period (which is the head that must be used in setting the prime rate) is not as great as the average operating head for the entire period of record. The two common assumptions concerning the latter—(1) that average head equals gross head less one-third maximum draw-down, and (2) that average head equals gross head less draw-down corresponding to 50% useful storage capacity—are satisfactory for the purpose for which they were intended; that is, for determining the proper installation head for the turbines. When applied to duration-curve or mass-curve studies of the critical period they will indicate prime rates higher than could actually be maintained with the useful storage assumed.

Table 2 was compiled by the writer to aid in the selection of average head. As this head is a function both of the shape of the reservoir and of

TABLE 2.—RATIO OF AVERAGE DRAW-DOWN DURING CRITICAL PERIOD TO MAXIMUM DRAW-DOWN

Reservoir No.	Gross head, in feet (1)	Maximum draft, in feet (2)	Average Q, in cubic feet per second (3)	Kilowatts (4)	Average head, in feet (5)	Average draw-down (6)	Ratio (7)	Length of period, in months (8)
1........	193.5	51.1	6 487	72 000	163.6	29.9	0.585	9
2........	104.3	31.3	7 297	43 000	86.9	17.4	0.556	9
3........	109.2	29.7	3 028	18 700	91.0	18.2	0.613	9
4........	435.5	164.0	2 510	61 000	358.0	77.5	0.473	18
5........	99.0	21.0	3 087	18 900	90.3	8.7	0.414	16

the distribution of flows during the critical period, nothing more than a general rule can be laid down, the actual selection in any particular case being made on the basis of similarity of conditions with those of some plant already studied analytically.

The following will explain the entries in Table 2: Column (1) represents the static head at full reservoir; Column (2) is the draw-down at the end of the critical period; values in Column (3) were computed from the analyti-

cally determined, month-by-month discharge requirement to maintain the rate of Column (4); Column (4) is the rate, in kilowatts, maintained during the critical period; Column (5) is the head through which the average quantity (Column (3)) would have to pass to maintain the rate listed in Column (4) at 80% over-all plant efficiency; Column (6) is the difference of Column (5) from Column (1); Column (7) is the ratio of Column (6) to Column (2); and, Column (8) represents the time from the beginning of reservoir draw-down until the maximum draw-down occurs.

From these and similar observations the writer has concluded that the average head at a reservoir plant during the critical period should usually be taken as the gross head less 55 to 60% of the maximum draw-down, for purposes of mass-curve or hydrograph studies.

In Reservoirs Nos. 4 and 5, regulation extending over more than a year was involved. It happened that, in both cases, the demands during the first year were relatively small, the reservoir maintaining a fairly high level throughout the year, but failing to fill on account of low flows during the usual peak months. The average draw-down for the entire period is thus proportionately less, and the average head higher, than for the first three cases. Had the maximum demands come in the first year of draft the conditions would have been reversed.

L. T. Guy,[7] Assoc. M. Am. Soc. C. E. (by letter).[7a]—An interesting manner of using the duration curve, in solving the various problems pertaining to storage capacities, has been developed in this paper. As stated, this method has its particular application in the case of preliminary investigations, and the authors stress its advantages over "cut-and-try" hydrographic studies.

However, for the determination of storage capacities and the capacity of storage plants, the writer cannot see that the "duration curve" has any advantages over the "mass curve." On the contrary, the latter method appears to have some distinct advantages. As stated by the authors, their duration curve must be prepared from the flow records for a complete cycle, from full reservoir to full reservoir. The determination of this period necessitates the plotting of the mass curve and, having this curve, the duration curve is only an unnecessary complication.

A superior method of plotting mass curves, which was originally brought to the writer's notice by a Norwegian engineer, is shown in Fig. 7. This method has been found to have several advantages, and is thought to be not generally well known. Fig. 8 is plotted from the mass curve of a stream for which records are available over a period of thirty-nine years, including two or three periods of extremely dry flow. It will be noted that these curves indicate the storage capacity necessary to maintain any desired regulated flow.

It is clear that Fig. 8 can be used, not only for the stream to which these records refer, but also for other streams that have approximately the

[7] Asst. to Civ. Engr., State Electricity Comm. of Victoria, Melbourne, Victoria, Australia.

[7a] Received by the Secretary August 7, 1932.

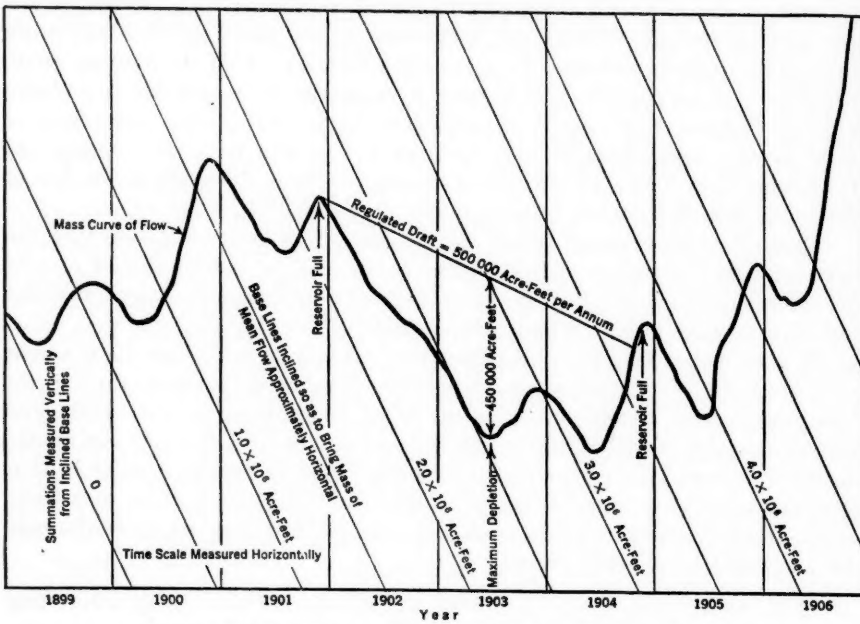

FIG. 7.—MASS CURVE OF STREAM COVERING CRITICAL PERIOD.

same flow characteristics, where records over long periods may be lacking. In the State of Victoria, Australia, many of the streams have remarkably

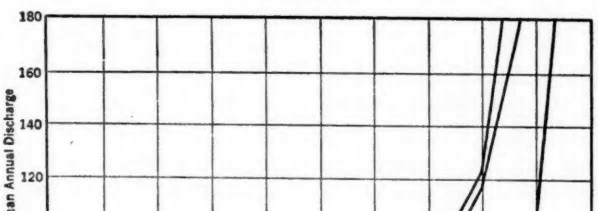

FIG. 8.—REGULATION CURVES FOR MITRA RIVER.

similar flow characteristics, and may readily be grouped together in this manner, when making preliminary investigations. This means of ascer-

taining the necessary storage capacity appears to be more direct than that of plotting duration curves.

Furthermore, when complications arise, such as head varying with "draw-down" of storage, load factors varying with seasons, etc., it appears much simpler to make the necessary adjustments to the mass curve rather than resort to the duration-curve method. In the case of the mass curve, the effects of each step in any such adjustment are obvious, while for the duration curve, they would not appear to be so simple.

While the writer prefers the mass-curve method for the determination of storage problems, the fact remains that the duration curve may also have its place in this regard, and is undoubtedly essential when dealing with "run-of-stream" plants. In many cases, a combination of the two methods provides an excellent means of solving the problem.

JOEL D. JUSTIN,[8] M. AM. SOC. C. E. (by letter).[8a]—Over a wide range of problems involving load, steam and hydro-electric power, storage, stream flow, and various combinations of these factors, the duration curve is a time-saving and convenient tool, worthy of more general acceptance than it has received. The interesting paper by Messrs. Hickox and Wessenauer should serve to make a larger number of engineers familiar with the advantages of these curves. Although duration curves have been used by some engineers in connection with problems very similar to those outlined by the authors, it is probable that the precise manner of application which they present is quite novel.

Economic considerations are tied up so intimately at every step with studies of this kind that it is impracticable to eliminate "cut and try" methods entirely. Thus, after making the various assumptions required in such a study, computations are made, curves are drawn, and a "required" storage or installation is determined. The installation or storage determined by the methods outlined by the authors may not prove to be economically feasible. For this reason, it is well to keep constantly in mind the economic factors at each step in the computations.

In considering the question of storage for hydro-electric power, it is usually assumed that if relatively cheap storage is available, it is highly desirable to be able to turn out as much primary energy as possible. With modern interconnected power systems served by both steam and hydro-electric plants, sufficient storage to provide a large amount of primary energy might be an economic waste. Practically the entire function of storage in such systems is to provide enough primary energy to make the capacity of the hydro-electric plants "firm capacity"; that is, to make it capable of performing the same function as alternative steam capacity.

Fig. 9 shows a duration load curve for the peak December week of a typical power company, having a peak load of 100 000 kw. The load factor of the curve is approximately 53 per cent. To assume a simple case, there is only one hydro-electric plant in this system having a capacity of 30 000 kw.,

[8] Hydro-Elec. Engr., U. G. I. Co., Philadelphia, Pa.

[8a] Received by the Secretary August 2, 1932.

with a pond of sufficient size to provide weekly regulation, and there are also several steam plants. From this curve, it is found that by measuring down from the peak, 30 000 kw., the cross-hatched area containing 360 000 kw-hr. is cut off. If at the hydro-electric plant there are at least 360 000 kw-hr. available in a week of minimum December stream flow at the plant, then the entire 30 000-kw. installation will be firm capacity. Although the capacity

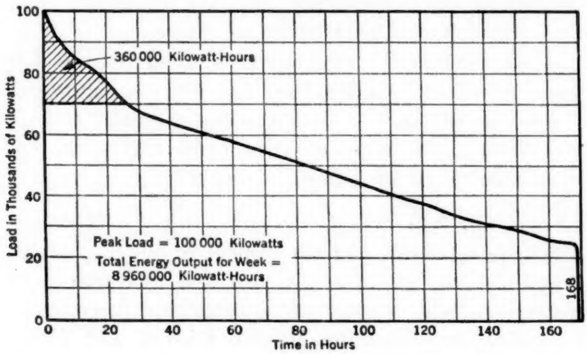

FIG. 9.—LOAD–DURATION CURVE, DECEMBER PEAK LOAD WEEK FOR A TYPICAL REGIONAL POWER COMPANY.

factor of the hydro-electric plant for the week will be only 7%, it has done exactly the same work that alternative steam capacity could do.

Additional primary energy in excess of the 360 000 kw-hr. required, would have had only the value of marginal steam-generated energy. However, additional storage might permit of the installation of more firm capacity later. The principal value of storage in a modern power system is the creation of firm hydro-electric capacity, and in making studies similar to those described by the authors this fact should be constantly kept in mind.

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DISCUSSIONS

WIND-BRACING IN STEEL BUILDINGS

SECOND PROGRESS REPORT OF SUB-COMMITTEE NO 31, COMMITTEE ON STEEL OF THE STRUCTURAL DIVISION

Discussion

BY JOHANNES SKYTTE, ASSOC. M. AM. SOC. C. E.

JOHANNES SKYTTE,[66] ASSOC. M. AM. SOC. C. E. (by letter).[66a]—According to the report, the secondary effect of the lengthening and shortening of columns due to their direct stresses should be investigated for tall and narrow buildings (see under "(2) A Method of Analysis of Shallow Bracing Systems," and Conclusion (4)). For practical reasons it would scarcely be possible to proportion the different members so that, theoretically, the joints at any floor were maintained in a straight line. However, any distortion tending to bring the joints out of a straight line will produce such internal shear forces as will work against this deviation and will have a great reducing effect. The writer believes that, provided the engineer—with due consideration of practical limits—will proportion the different members, a straight-line variation for the joints can practically be assumed. Then, the analysis will be as follows.

The lengthening and shortening of the columns will produce vertical shears in the girders, but no shears in the columns. The shape of the elastic line of the deformed bent will be as shown in Fig. 34. An inspection of Figs. 34 and 35 shows that:

(1) The frame will be displaced horizontally;
(2) The girder moments have signs opposite to those produced by the shears from the lateral load, and, consequently, reduce these moments and can be neglected;

NOTE.—The report of Sub-Committee No. 31, Committee on Steel, of the Structural Division, on Wind-Bracing in Steel Buildings, was presented at the meeting of the Structural Division, New York, N. Y., January 21, 1932, and published in February, 1932, *Proceedings*. Discussion on this report has appeared in *Proceedings* as follows: May, 1932, by Messrs. L. J. Mensch, Robins Fleming, Rudolph P. Miller, C. M. Goodrich, Albert Smith, Hugh L. Dryden, L. E. Grinter, P. L. Pratley, Frederick Martin Weiss, and A. J. Hammond; August, 1932, by Messrs. David Cushman Coyle, S. P. Wing, A. H. Finlay, J. D. Gedo, V. A. Vanoni and M. P. White, and H. V. Spurr; and September, 1932, by Messrs. O. G. Julian, and Jacob Feld.

[66] Asst. Hydr. Engr., Hetch Hetchy Project, San Francisco, Calif.

[66a] Received by the Secretary August 15, 1932.

(3) The columns will bend as long independent cantilevers from the moments introduced by the girder shears, X_a, X_b, X_c,..., and, consequently, have a bending moment equal to zero at the top of the building and increasing to a maximum at the bottom; and,

(4) The vertical displacement of the joints consists of two parts (see Fig. 35), namely, from the rotation and $2\epsilon_c$ from cantilever deflection, thus:

$$\epsilon = 2\,(\epsilon_\phi + \epsilon_c) = 2\left(\phi\,\frac{s_a}{2} + \frac{X_a\,s_a^3}{24\,E\,I_a}\right) = \phi\,s_a + \frac{X_a\,s_a^3}{12\,E\,I_a} \quad \ldots\ldots(10)$$

in which (in addition to the notation of the report), ϵ = vertical displacement of joints; ϕ = change in slope of tangent to the elastic line; X = girder shears; and s = span of girders.

Assume that the vertical displacement, ϵ, is caused by rotation only. This will almost be the case as the girder shears must consist of small forces and as the girders are very stiff compared to the columns as long cantilevers.

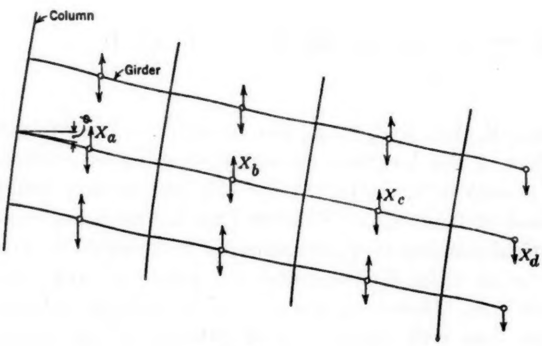

FIG. 34.

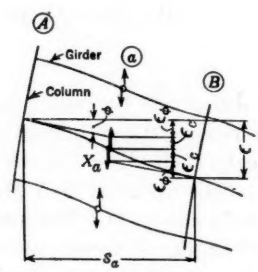

FIG. 35.

Let ρ = radius of curvature for the elastic line of columns; x = distance from bottom of building to point under consideration; and y = horizontal displacement of point under consideration. Then, the well-known equations for the elastic line give:

$$\frac{d^2 y}{d x^2} = \frac{M}{E\,I} = \frac{1}{\rho} \quad \ldots\ldots(11)$$

From Fig. 36,

$$\frac{d\,\phi}{d x} = \frac{1}{\rho} \quad \ldots\ldots(12)$$

and for $\epsilon_c = 0$, Equation (10) gives,

$$\epsilon = \phi\,s_a \quad \ldots\ldots(13)$$

It can easily be shown that, approximately,

$$\epsilon = C\,\frac{w_l\,f_{sg}\,(2\,H - x)\,x}{w_g\,E\,4 B} \quad \ldots\ldots(14)$$

In Equation (14), C is always less than unity; however, in this case, let $C = 1$; w_l = lateral load on bent per foot of height of building; w_g = gravity

load on outside column per foot of height of building; f_{sg} = average compressive stress in outside column from gravity load; H = height of building; and B = width of building. Substituting ϵ from Equation (13) in Equation (14), $\phi = \frac{w_l f_{sg} (2 H - x) x}{w_g E 4 B s_a}$; and by differentiation:

$$\frac{d\phi}{dx} = \frac{w_l f_{sg} (H - x)}{w_g E 2 B s_a} \quad \text{.....................(15)}$$

From Equations (11), (12), and (15),

$$\frac{d^2 y}{d x^2} = \frac{M}{EI} = \frac{1}{\rho} = \frac{w_l f_{sg} (H - x)}{w_g E 2 B s_a} \quad \text{...............(16)}$$

Integrating;

$$y = \frac{w_l f_{sg} (3 H x^2 - x^3)}{w_g E 12 B s_a} \quad \text{.....................(17)}$$

which is the equation for the elastic line.

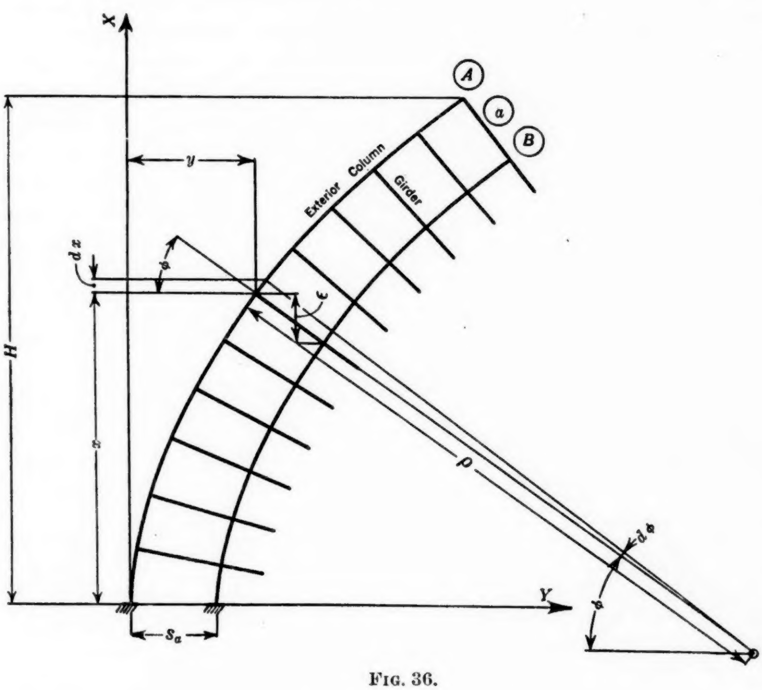

FIG. 36.

As an example, consider a building 500 ft. tall, having a width of 60 ft. For f_{sg} = 12 500 lb. per sq. in., s_a = 15 ft.; w_l = 450 lb. per ft.; and w_g = 4 000 lb. per ft. Equation (14) gives, $\epsilon_{max.} = \frac{450 \times 12\,500 \times 500^2 \times 12}{4\,000 \times 30 \times 10^6 \times 4 \times 60}$ = 0.59 in.

The radius of curvature at the bottom of the building is found from Equation (16); thus: $\rho = \dfrac{4\,000 \times 30 \times 10^6 \times 2 \times 60 \times 15}{450 \times 12\,500 \times 500} = 76\,500$ ft.

If the heaviest column in the bottom story is a 16-in., 912-lb. **H**-column $I = 23\,948$; $s = 1\,901.4$; and, consequently,

$$M = \frac{30 \times 10^6 \times 23\,948}{12 \times 76\,500} = 780\,000 \text{ in-lb.}$$

and,

$$f_s = \frac{780\,000}{1\,901.4} = 410 \text{ lb. per sq. in.}$$

The writer realizes the approximate nature of the foregoing analysis, and that the stresses caused by the secondary effect might be somewhat higher than shown in the example (due to slight deviation from a straight line of joints at the same floor). However, by using proper judgment in the design the wisdom of Nature will do its share to make these stresses insignificant.

APPLICATIONS FOR ADMISSION AND FOR TRANSFER

The Constitution provides that the Board of Direction shall elect or reject all applicants for *Admission* or for *Transfer*, and, in order to determine justly the eligibility of each candidate, the Board must depend largely upon the Membership for information.

This list is issued to members in every grade for the purpose of securing all such available information, and every member is urged to scan carefully each monthly list of candidates and to furnish the Board with data in regard to any applicant which may aid in determining his eligibility. It is the *Duty* of all *Members* to the *Profession* to assist the *Board* in this manner.

It is especially urged, in communications concerning applicants, that a *Definite Recommendation as to the Proper Grading in Each Case* be given, inasmuch as the grading must be based upon the opinions of those who know the applicant personally, as well as upon the nature and extent of his professional experience. If facts exist derogatory to the personal character or to the professional reputation of an applicant, they should be promptly communicated to the Board. *Communications Relating to Applicants are considered by the Board as Strictly Confidential.*

The Board of Direction will not consider the applications herein contained from residents of North America until the expiration of thirty (30) days, and from non-residents of North America until the expiration of ninety (90) days from October 15, 1932.

MINIMUM REQUIREMENTS FOR ADMISSION

Grade	General Requirement	Age	Length of Active Practice	Responsible charge of work
Member	Qualified to design as well as to direct important work	35 years	12 years*	5 years
Associate Member	Qualified to direct work	27 years	8 years*	1 year
Junior	Qualified for sub-professional work	20 years†	4 years*	
Affiliate	Qualified by scientific acquirements or practical experience to co-operate with engineers	35 years	12 years*	5 years
Fellow	Contributor to the permanent funds of the Society			

* Graduation from a school of engineering of recognized reputation is equivalent to 4 years active practice.

† Membership ceases at age of 33 unless transferred to higher grade.

The fact that applicants give the names of certain members as references does not necessarily mean that such members endorse.

FOR ADMISSION

ANSON, EDWARD HIRAM, New York City. (Age 29). Asst. Engr. with Gibbs & Hill, Cons. Engrs. Refers to E. H. Cameron, E. R. Hill, H. W. Hudson, O. G. Julian, E. L. Moreland, J. H. O'Brien.

BOLINAS, ANDRES, Jr., Legaspi, Albay, Philippine Islands. (Age 34). Civ. Engr., Surveyor and Contr. Refers to H. V. Campbell, M. Cruz, S. Garmezy, V. Segura, E. A. Silagi, A. R. Webb.

BOYD, GEORGE EDWARD, New York City. (Age 41). Sales Engr. and New York Dist. Mgr., Wailes Dove-Hermiston Corporation. Refers to W. W. Brush, D. W. Coe, J. E. Garratt, M. I. Killmer, R. Ridgway, J. F. Sanborn, J. B. Snow.

COLANTINO, THOMAS, Mamaroneck, N. Y. (Age 25). Asst. Supt., D. Colantino Corporation. Refers to E. R. Cary, L. W. Clark, H. B. Compton, T. R. Lawson, C. P. Rumpf, H. O. Sharp.

DAVIS, PAUL WESLEY, Pittsburgh, Pa. (Age 29). With The Wichert Continuous Bridge Corporation. Refers to J. T. Campbell, D. E. Davis, F. M. McCullough, H. B. Miller, G. S. Richardson, C. B. Stanton.

DEWEESE, OMER LYNN, Evansville, Ind. (Age 28). Chf. of Survey Party, Indiana State Highway Comm., Indianapolis, Ind. Refers to H. N. Barnard, A. A. Cummins, C. M. DuBois, W. J. Titus, N. F. Williams.

DOWNWARD, PAUL HOLLINGSWORTH, East Orange, N. J. (Age 28). With Hart & Early Co., Contrs. and Engrs., New York City. Refers to F. E. Cudworth, J. J. Durfee, C. Goodman, R. W. Greenlaw, C. T. Schwarze.

FREYBERG, WOLDEMAR OSCAR, Ann Arbor, Mich. (Age 53). Research Engr., Eng. Research Dept., Univ. of Mich. Refers to B. A. Bakhmeteff, J. H. Cissel, E. L. Eriksen, W. S. Housel, F. N. Menefee, W. C. Sadler, R. H. Sherlock, J. F. Stevens, J. S. Worley.

FRIEDLANDER, MORTIMER ALTMAN, Brooklyn, N. Y. (Age 25). Refers to C. L. Eckel, E. W. Raeder.

GALLAGHER, WILLIAM JOSEPH, Davenport, Iowa. (Age 23). Engr. with Blunk & Joehnke. Refers to A. H. Holt, J. W. Howe, R. B. Kittredge, B. J. Lambert, F. T. Mavis, F. A. Nagler.

GEILE, WILFRED GEORGE, Raleigh, N. C. (Age 37). Associate Prof. and Head, Dept. of Constr. Eng., North Carolina State Coll. of Agriculture and Eng. Refers to H. C. Bird, C. T. Bishop, C. L. Mann, W. C. Riddick, C. J. Tilden, J. C. Tracy, H. Tucker.

GORDANIER, JOHN WEDGWOOD, Boulder City, Nev. (Age 22). Laboratory Asst., Concrete Testing Laboratory, U. S. Bureau of Roads. Refers to M. K. Snyder, J. G. Woodburn.

GRUNWELL, GILBERT BUTTERFIELD, Punta Gorda, Fla. (Age 28). Rodman, U. S. Geological Survey. Refers to S. A. Becker, J. M. Canals, R. J. Fogg, M. O. Fuller, T. Human, Jr., W. L. Wilson.

HARRIS, HOMER HENRY, Jr., Alexandria, La. (Age 21). Refers to B. W. Pegues, I. W. Sylvester.

KAHAN, ARTHUR, Brooklyn, N. Y. (Age 23). Refers to R. E. Goodwin, F. O. X. McLoughlin.

KELLEY, ARTHUR PAUL, New York City. (Age 22). Refers to W. K. Hatt, S. C. Hollister, R. B. Wiley.

LANGER, HENRY JULIUS, Minneapolis, Minn. (Age 23). Refers to F. Bass, A. S. Cutler.

LINCK, MICHAEL KERWIN, Syracuse, N. Y. (Age 24). Refers to E. F. Berry, L. Mitchell, S. D. Sarason.

MONONOBE, NAGAHO, Toshima, Tokyo, Japan. (Age 44). Director, Research Office, Experiment Station of Public Works, Japanese Govt.; also Prof., Civ. Eng. Dept., Tokyo Imperial Univ. Refers to M. Abe, K. Hirai, M. Kabashima, T. Kurashige, T. Shiraishi, A. N. Talbot, J. Yamaguchi.

SCHMIDT, GEORGE EDWARD, Pompey, N. Y. (Age 24). Refers to E. F. Berry, L. Mitchell, S. D. Sarason.

SHAHAN, MAXWELL LEE, Chattanooga, Tenn. (Age 23). Refers to R. P. Black, F. C. Snow.

SHAPIRO, SIDNEY, Wantagh, N. Y. (Age 28). Senior Park Engr., Long Island State Park Comm. Refers to J. M. Buckley, R. E. Coughlin, V. Gelineau, A. E. Howland, T. Saville, A. O. Smith, C. M. Spofford.

SHERMAN, STEWART ISRAEL, New York City. (Age 31). Asst. Engr., Board of Estimate and Apportionment, Office of Chf. Engr. Refers to D. W. Coe, L. Durham, W. J. Shea, M. Turk, A. S. Tuttle.

STEELE, JAMES POLK, Los Angeles, Cal. (Age 33). Gen. Supt. for Myers Bros., Gen. Contrs. Refers to J. C. Albers, H. A. Barnett, M. C. Blanchard, M. H. Fríncke, R. J. Hiller, O. S. Struthers, F. Thomas.

STRICKLER, WILBUR GOODWIN, Morgantown, W. Va. (Age 24). Refers to L. V. Carpenter, R. P. Davis, W. S. Downs.

UNDERWOOD, ERNEST JULIUS, Muscotah, Kans. (Age 21). Refers to L. E. Conrad, F. F. Frazier, M. W. Furr, L. V. White.

FOR TRANSFER

FROM THE GRADE OF ASSOCIATE MEMBER

CLARK, CHARLES SAMUEL, Assoc. M., Boston, Mass. (Elected July 9, 1923). (Age 40). With Massachusetts Bonding & Insurance Co. Refers to G. D. Andrews, A. W. Bowie, J. A. Giles, H. C. Hill, R. B. Hoadley, Jr., E. H. Prentice, W. E. Weller.

COTTON, HARRY EDMOND, Assoc. M., Boston, Mass. Elected Sept. 9, 1919. (Age 51). Sales Engr., New England Metal Culvert Co. Refers to H. Beal, J. A. Bruce, G. W. Craig, P. D. Sargent, G. E. Shafer, R. N. Towl, K. E. Vogel.

HALL, RUSSELL ALGER, Assoc. M., Schenectady, N. Y. (Elected Oct. 10, 1927). (Age 39). Assoc. Prof. of Civ. Eng., Union Coll. Refers to M. L. Enger, H. H. Jordan, M. S. Ketchum, H. Miller, A. N. Talbot.

HORTON, FREEMAN HUDSON, Assoc. M., Cincinnati, Ohio. (Elected July 16, 1928). (Age 35). Asst. Engr., Cincinnati Union Terminal Co. Refers to W. G. Brown, F. W. Dencer, O. E. Hovey, P. Jones, R. M. Miller, H. N. Rodenbaugh.

MOLTHER, FRANCIS RATTOONE, Assoc. M., Ancon, Canal Zone. Elected Aug. 18, 1930). (Age 39). Partner with F. J. Morales, C. E. Refers to J. H. Caton, 3d, H. R. Faison, R. W. Hebard, G. H. Hepburn, E. Jaen Guardia, E. J. Shaw.

NORRIS, ROBERT, Assoc. M., Ann Arbor, Mich. (Elected Nov. 27, 1917. (Age 45). Ayres, Lewis, Norris & May, Hydr. and Elec. Engrs. Refers to L. E. Ayres, J. H. Cissel, P. A. Fellows, G. H. Fenkell, L. M. Gram, M. Osgood, H. E. Riggs.

TATUM, ROBERT LEE, Assoc. M., Minden, La. Elected Junior, Nov. 8, 1909. Assoc. M., Nov. 4, 1914). (Age 49). Refers to C. W. Brown, P. E. Cunningham, J. R. Ellis, C. N. Kast, C. H. West.

WENDELL, EDWARD WILLOUGHBY, Assoc. M., Altamont, N. Y. (Elected March 13, 1917). (Age 47). Prin. Grade Separation Engr., New York State Dept. of Public Works. Refers to H. P. Barnes, R. G. Finch, F. S. Greene, L. G. Holleran, I. Van A. Huie, C. MacDonald, J. M. MacMartin, T. D. Pratt.

FROM THE GRADE OF JUNIOR

BARTON, HARRY, Jun., Bellevue, Pa. (Elected Feb. 24, 1931). (Age 32). Supt. and Gen. Mgr., Pittsburgh Suburban Water Service Co. Refers to A. W. Cuddeback, S. F. Newkirk, Jr., G. D. Norcom, A. T. Ricketts, W. H. Warnock.

COTTON, EDWIN ROWLAND, Jun., Williamsville, N. Y. (Elected Oct. 12, 1925). (Age 28). Cotton & Leupold, Cons. Engrs., Lancaster, N. Y. Refers to E. Devendorf, G. C. Diehl, G. F. Fisk, N. L. Nussbaumer, T. M. Ripley, H. T. Ware, E. A. Zeisloft.

EVANS, WILLIAM HAROLD, Jun., Hempstead, N. Y. (Elected March 12, 1923). (Age 32). Jun. Engr., New York & Queens Elec. Light & Power Co., Flushing, N. Y. Refers to H. B. Cleveland, A. L. Reeder, P. M. Van Camp, C. C. Vermeule, C. C. Vermeule, Jr.

FRICKER, FELIX OSCAR, Jun., Los Angeles, Cal. (Elected July 11, 1927). (Age 31). Asst. Engr., Quinton, Code & Hill-Leeds & Barnard, Engrs. Consolidated, Cons. Engrs. Refers to P. Bauman, L. C. Hill, R. A. Hill, C. T. Leeds, H. F. Peterson, R. J. Reed, M. E. Salsbury, F. Thomas.

The Board of Direction will consider the applications in this list not less than thirty days after the date of issue.

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* Director John R. Slattery died September 23, 1932.

AMERICAN SOCIETY OF CIVIL ENGINEERS

COMING MEETINGS

BOARD OF DIRECTION MEETINGS

January 16-17, 1933:
A Quarterly Meeting will be held at New York, N. Y.

ANNUAL MEETING

January 18, 19, 20, and 21, 1933

NEW YORK, N. Y.

January 18, 1933:
Morning.—Annual Meeting. Presentation of Medals and Prizes.
Afternoon.—Technical Meeting.
Evening.—President's Reception and Dinner.

January 19, 1933:
Morning.—Technical Division Sessions.
Afternoon.—Technical Division Sessions.
Evening.—Entertainment and Smoker.

January 20, 1933:
All-Day Excursion.

January 21, 1933:
Morning.—Inspection Trips.

The Reading Room of the Society is open from 9:00 A. M. to 5:00 P. M. every day, except Sundays and holidays; from May to September, inclusive, it is closed on Saturdays at 12:00 M.

Members, particularly those from out of town, are cordially invited to use this room on their visits to New York, to have their mail addressed there, and to utilize it as a place for meeting others. There is a file of 274 current periodicals, the latest technical books, and the room is well supplied with writing tables.